*Bienvenue!*

# ÉCOLE D'HIVER FRANCOPHONE EN APPRENTISSAGE PROFOND

5 - 9 mars 2018

IVADO
HEC Montréal
Polytechnique Montréal
Université de Montréal

MILA

# Optimisation pour les réseaux profonds

# École d'hiver du MILA

Nicolas Le Roux

Google Brain team

6/3/18

# Optimisation ?

- Un modèle est une fonction paramétrique
- On définit à la main la fonction d'erreur
- On cherche les paramètres qui minimisent l'erreur moyenne
- Entraîner un modèle = trouver les meilleurs paramètres = optimisation

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- Cet algorithme converge pour un “bon” $\alpha$ [Cau47].

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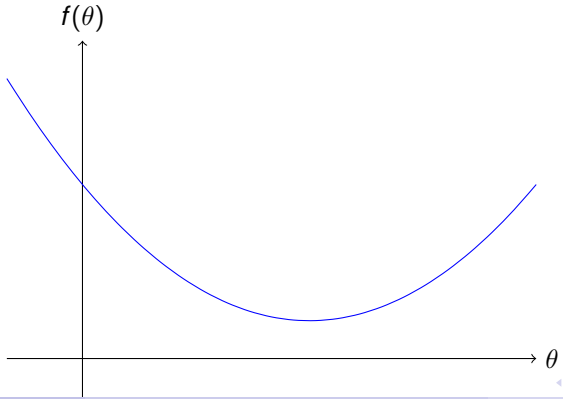

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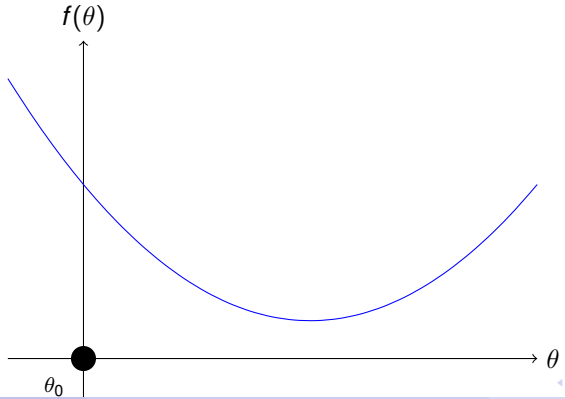

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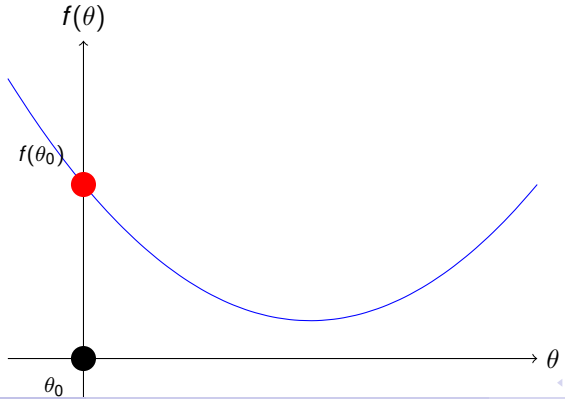

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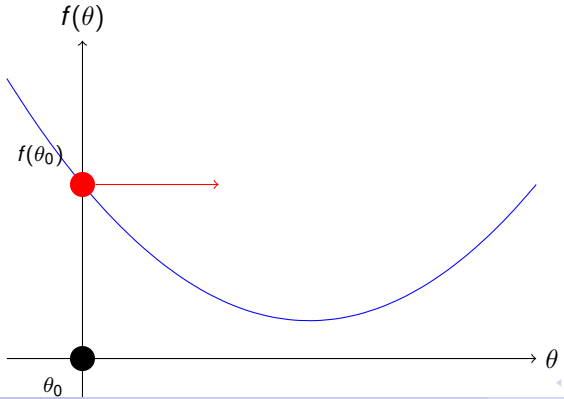

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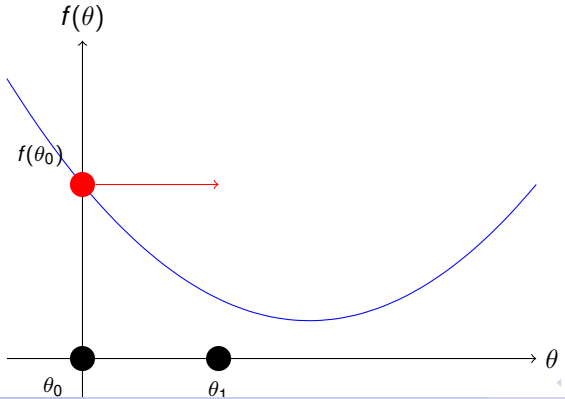

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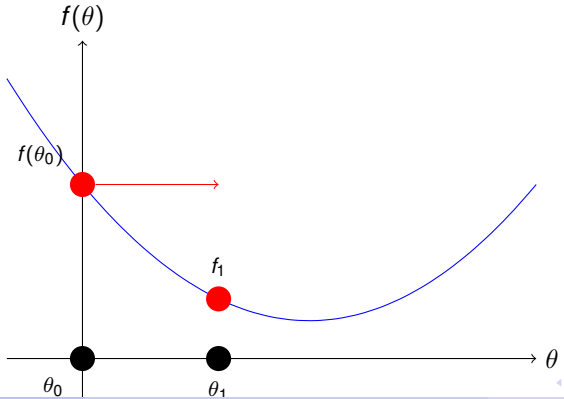

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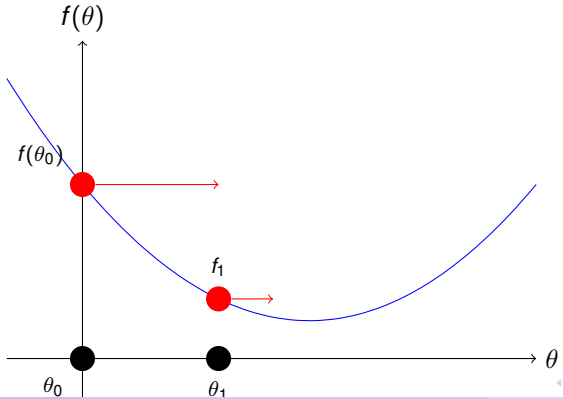

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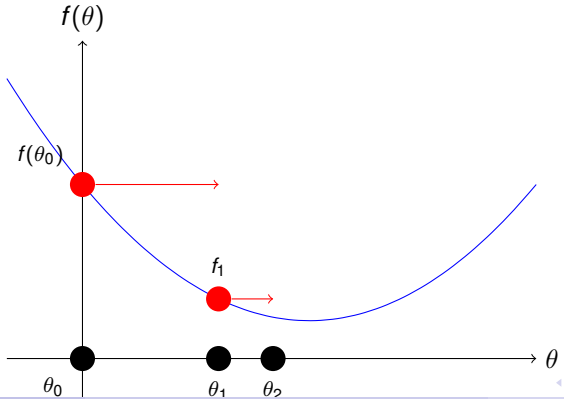

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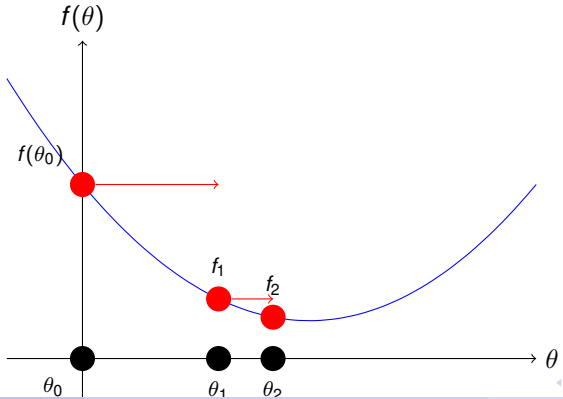

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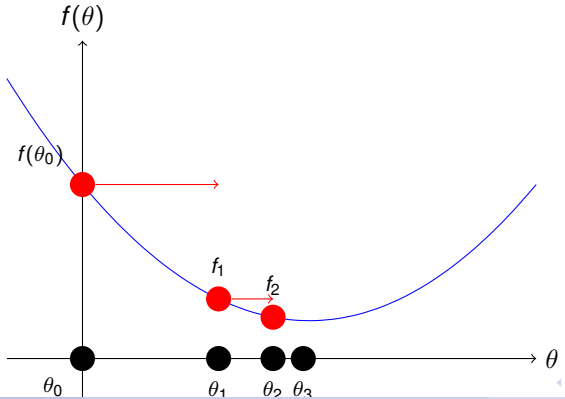

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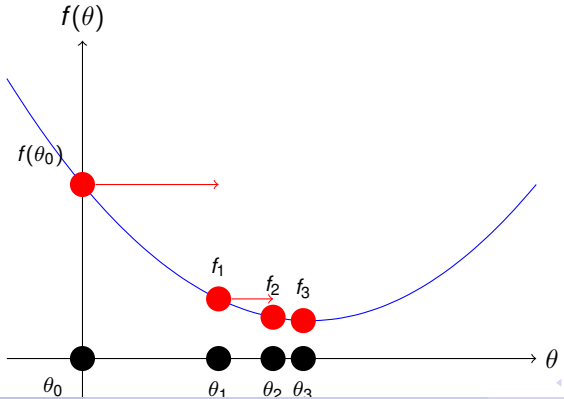

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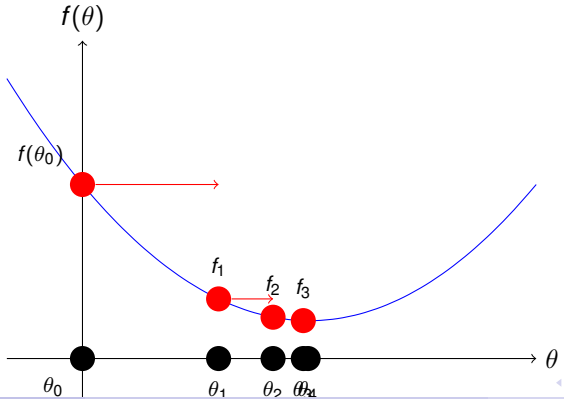

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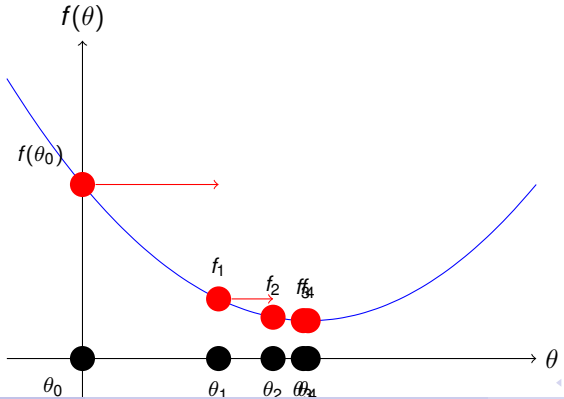

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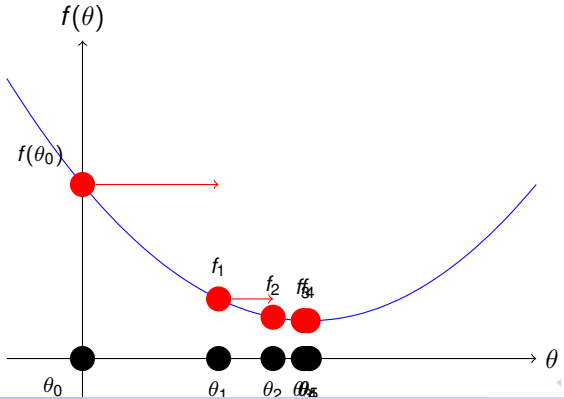

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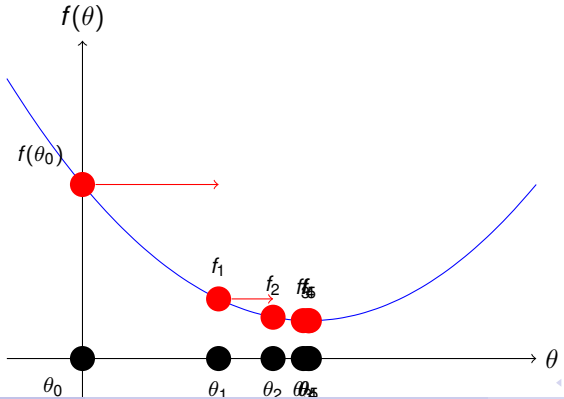

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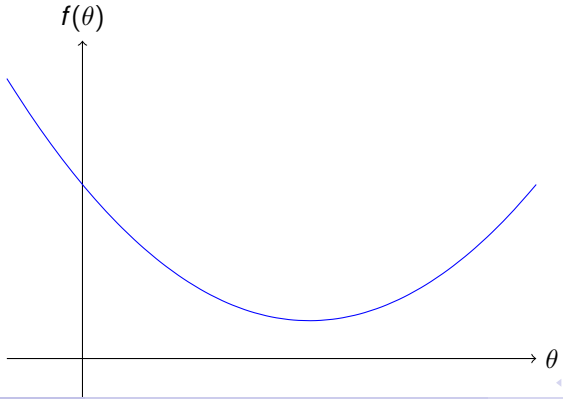

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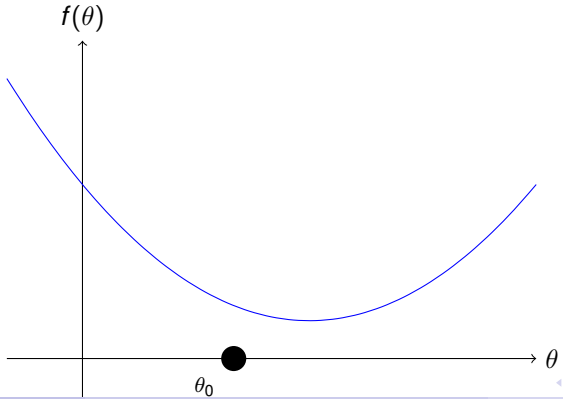

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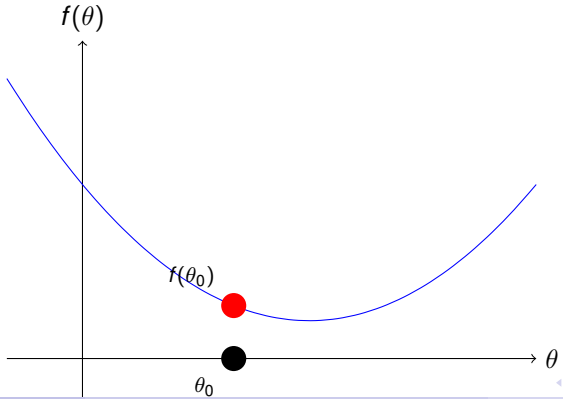

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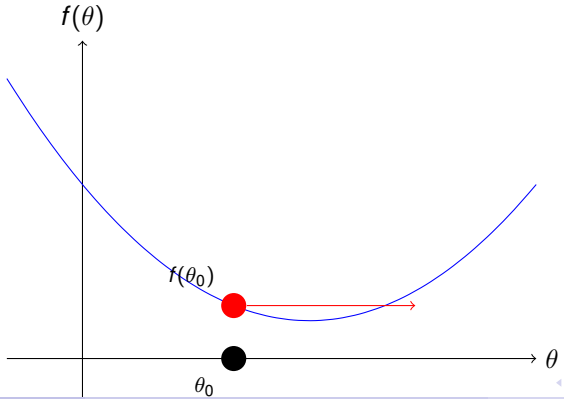

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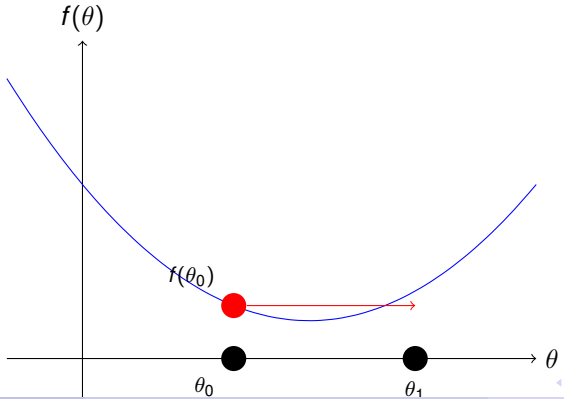

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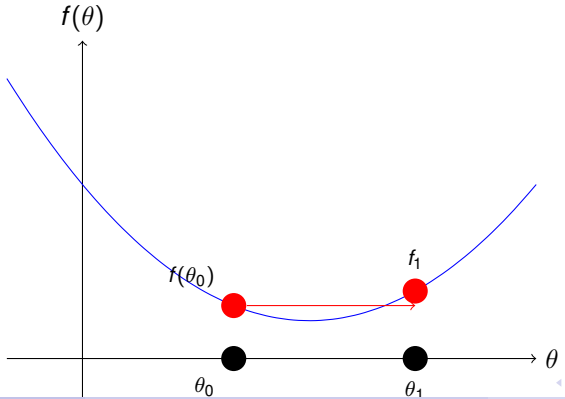

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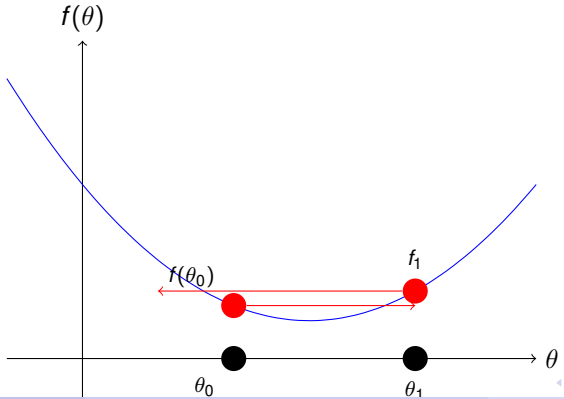

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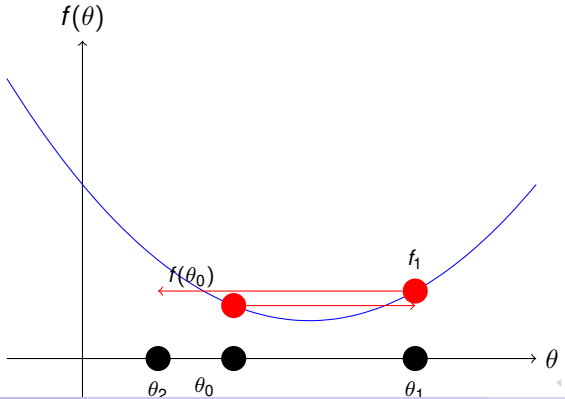

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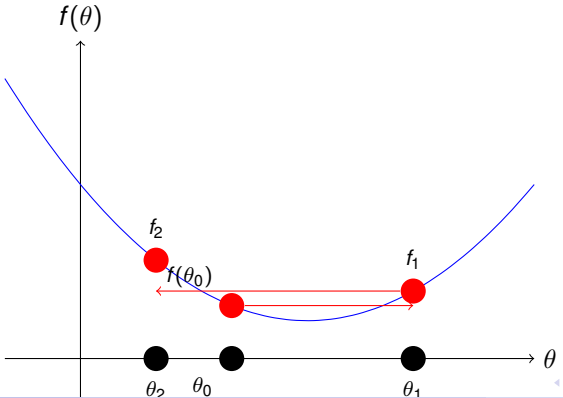

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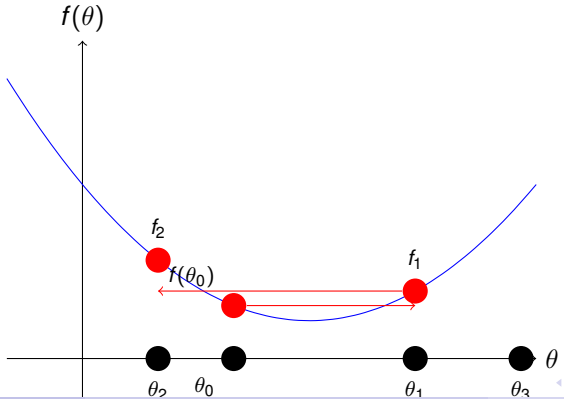

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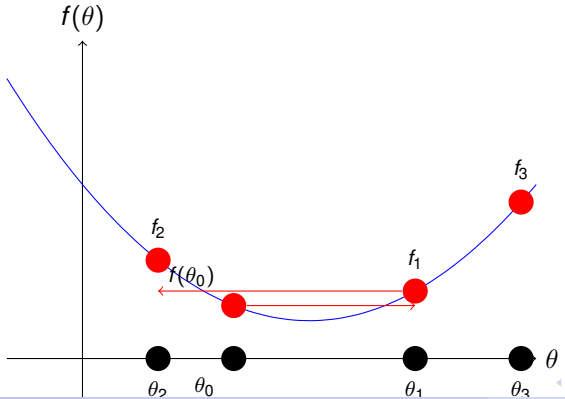

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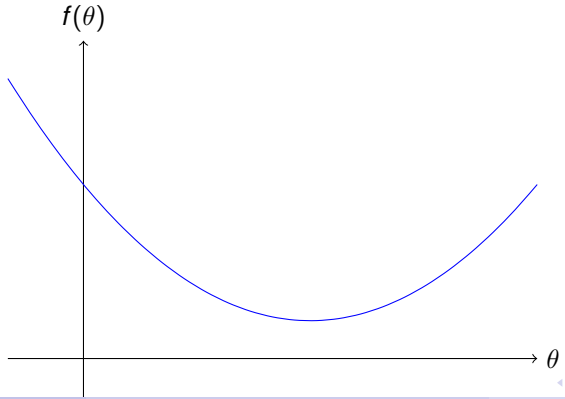

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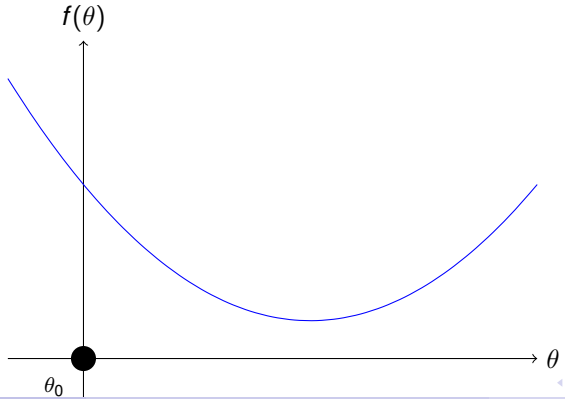

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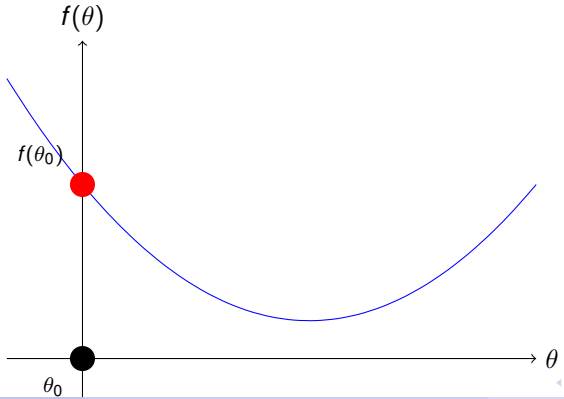

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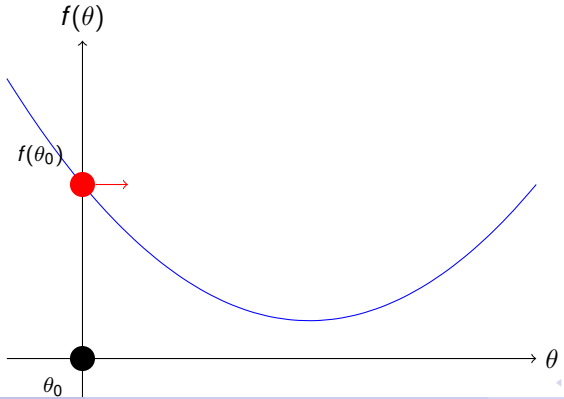

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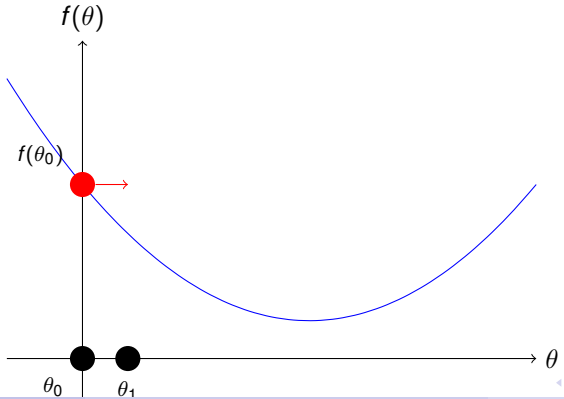

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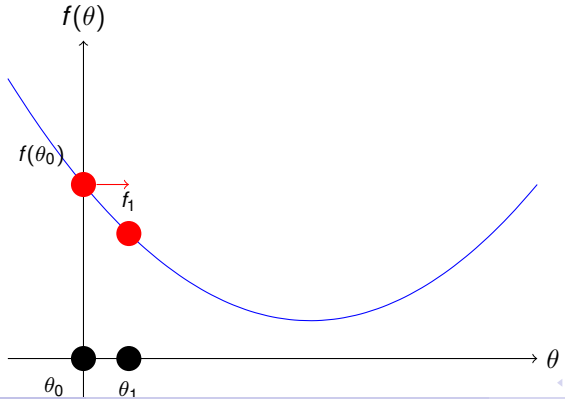

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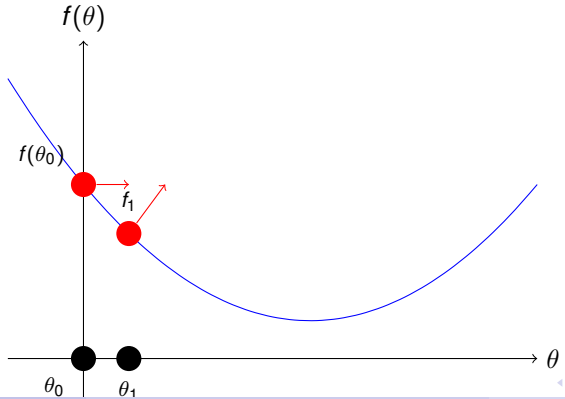

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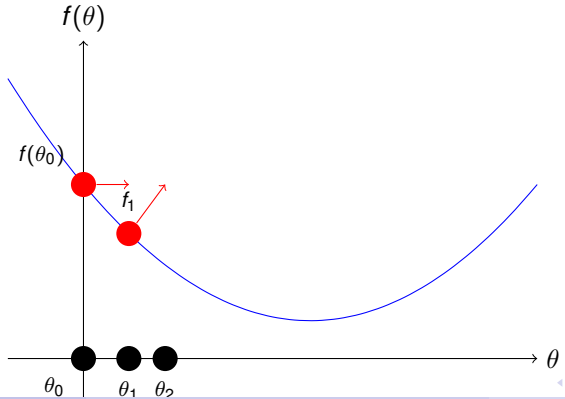

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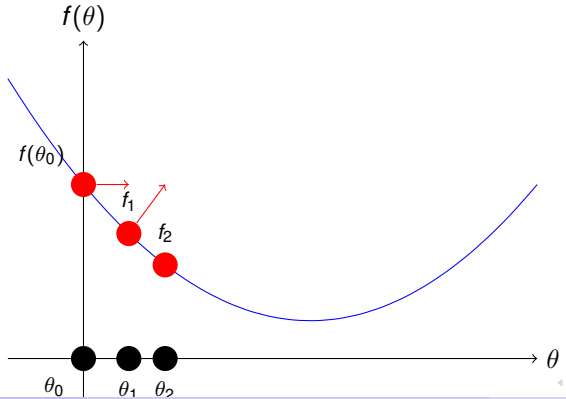

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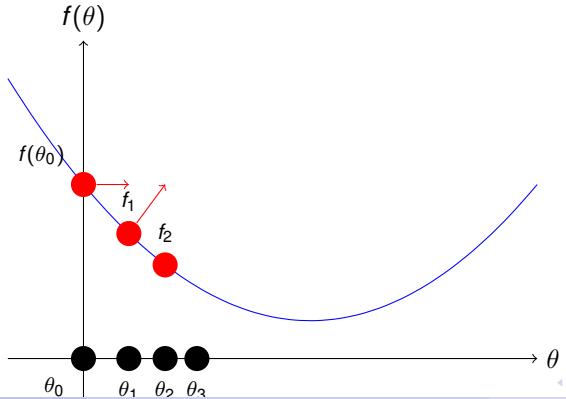

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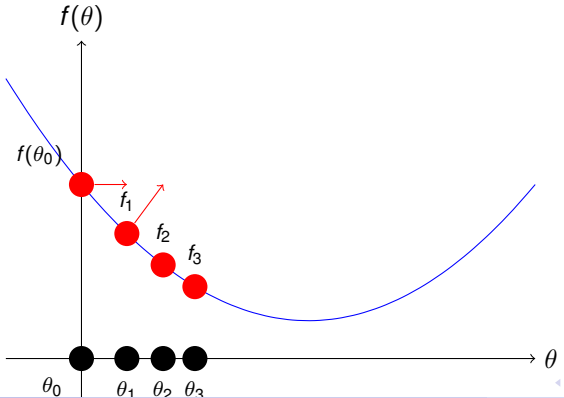

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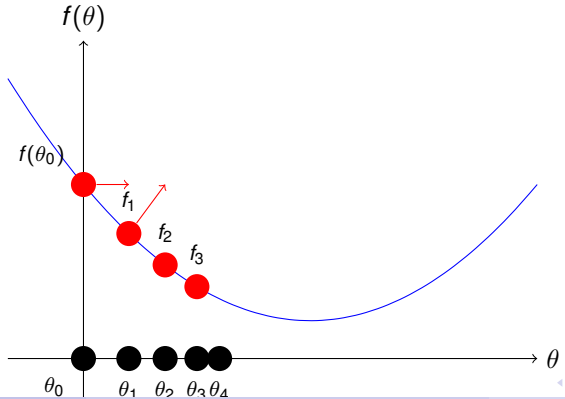

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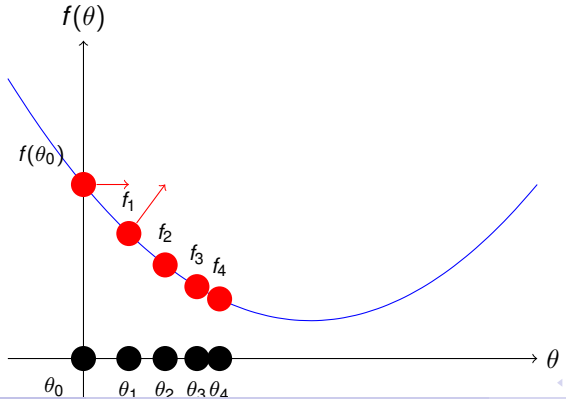

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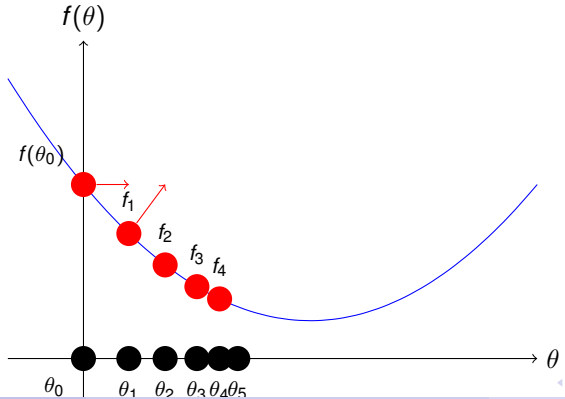

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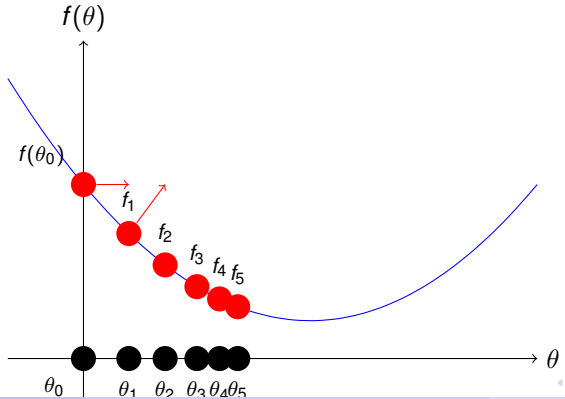

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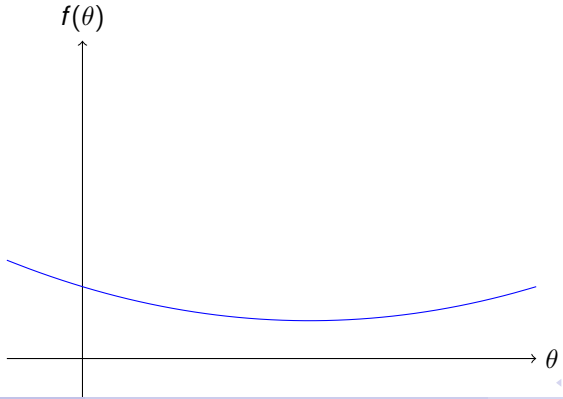

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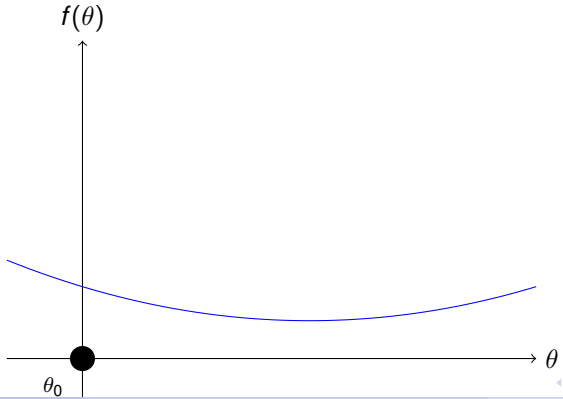

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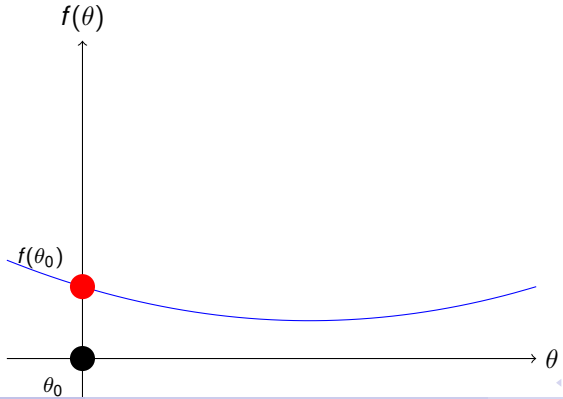

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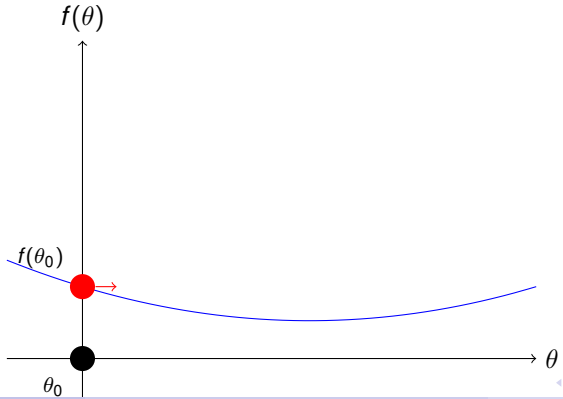

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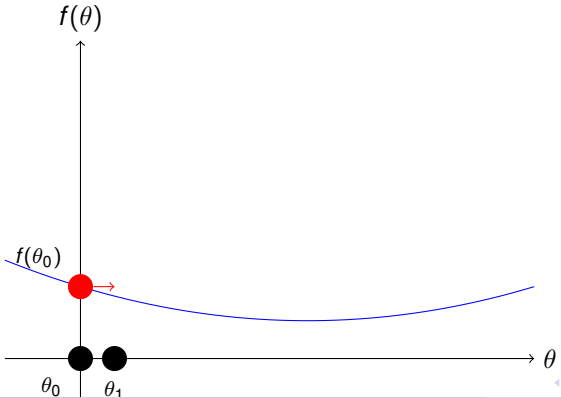

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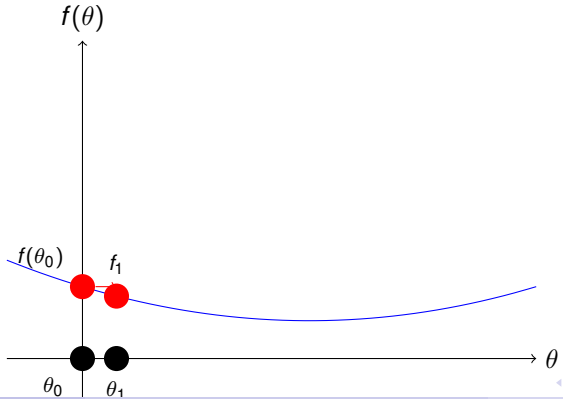

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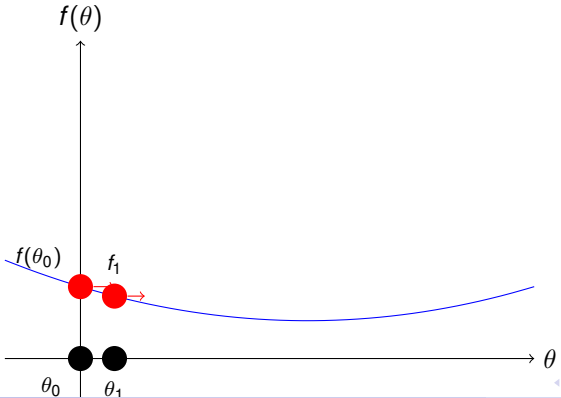

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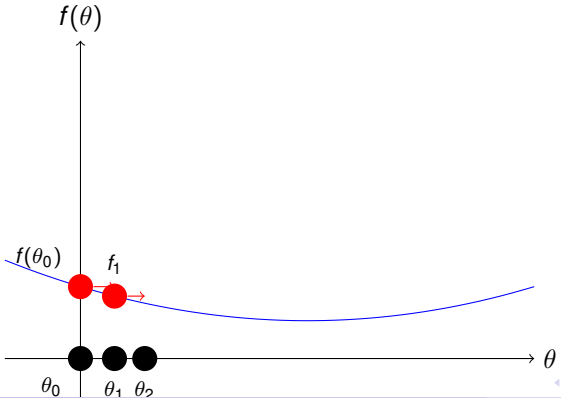

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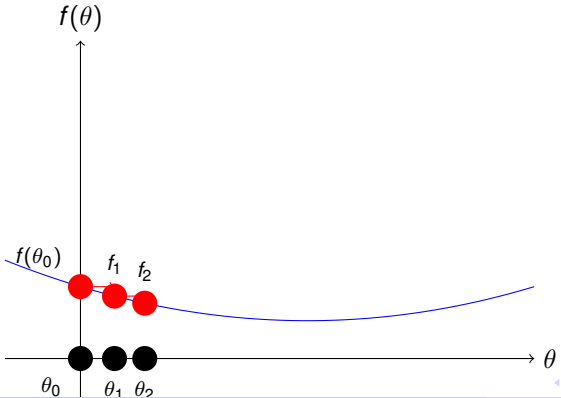

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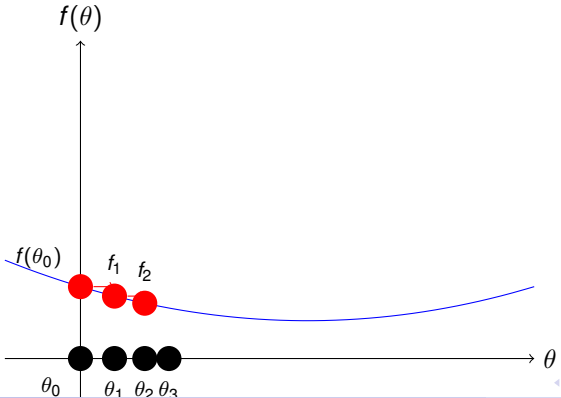

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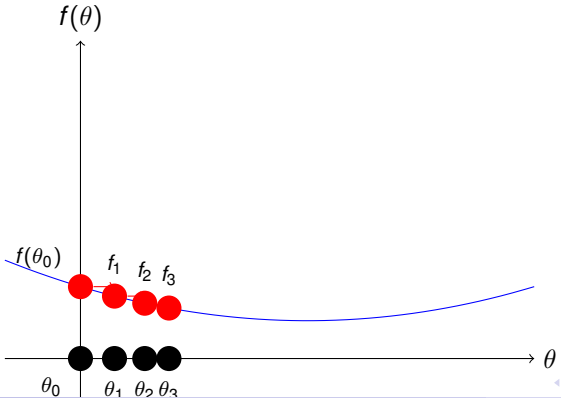

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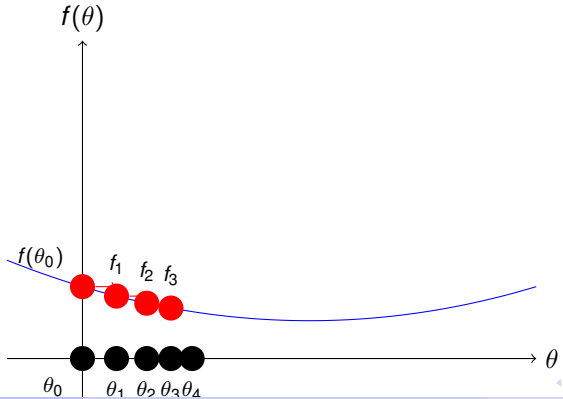

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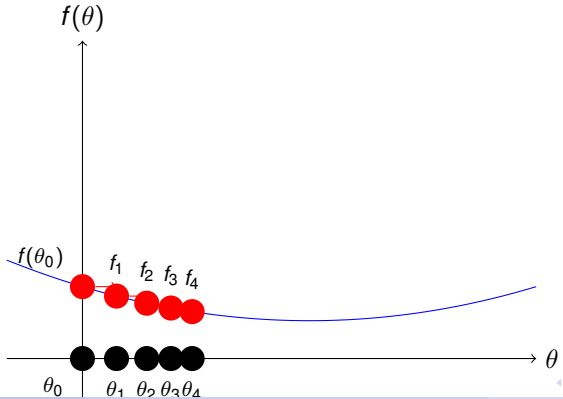

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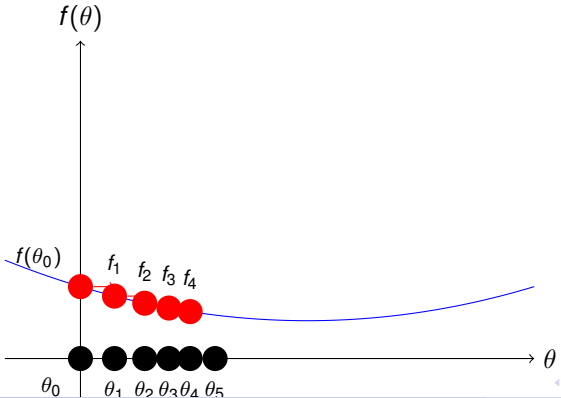

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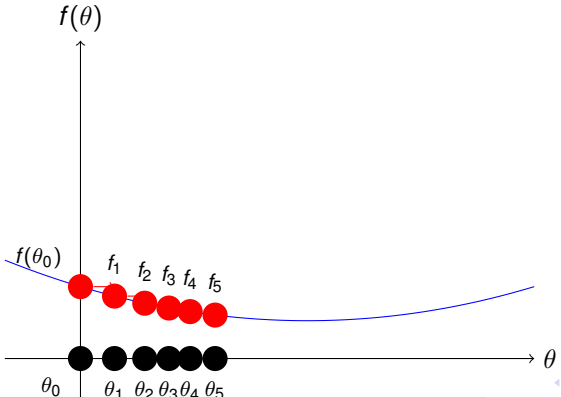

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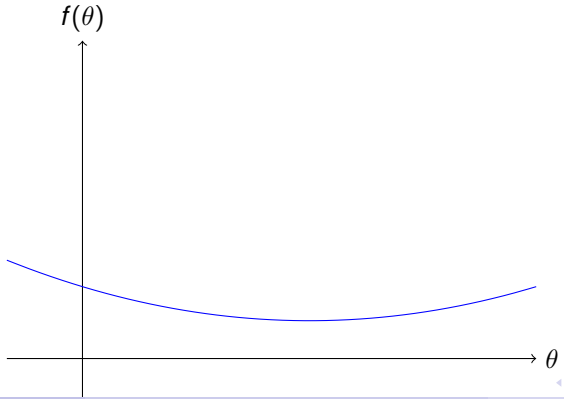

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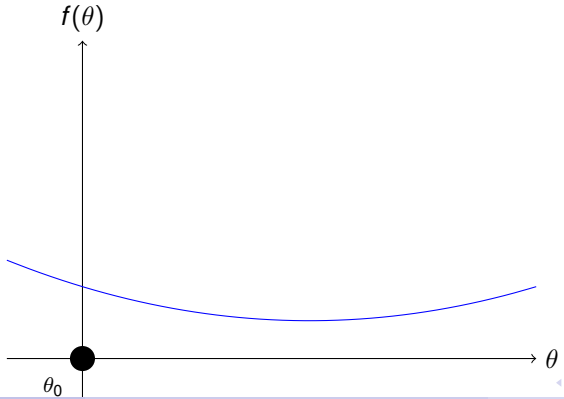

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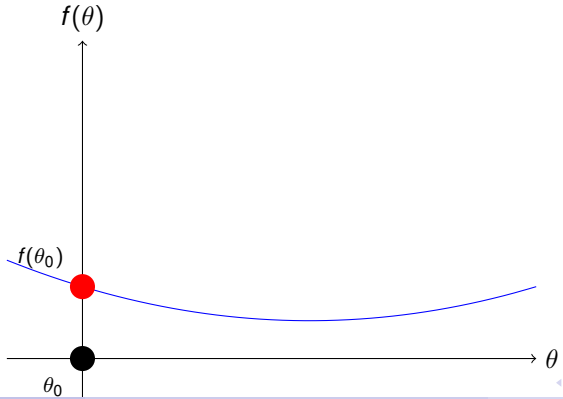

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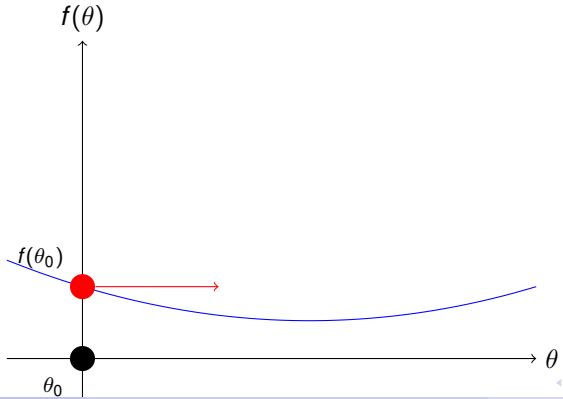

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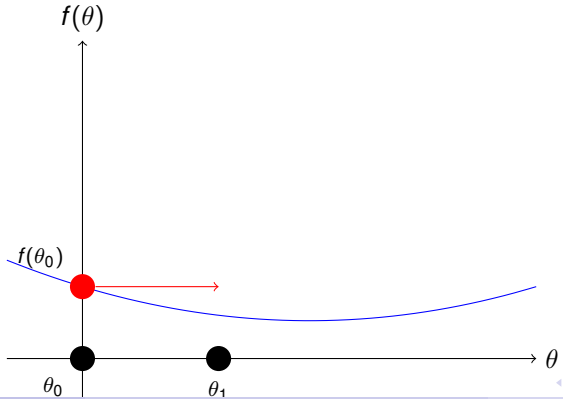

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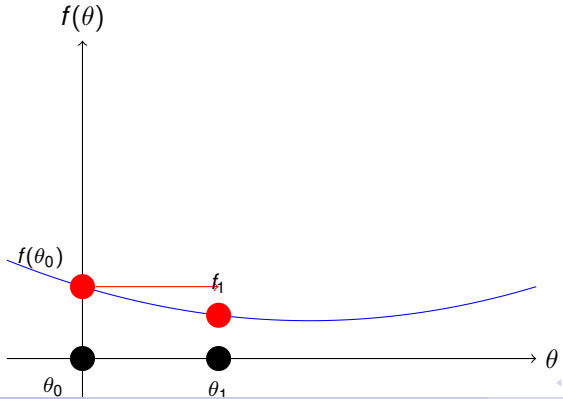

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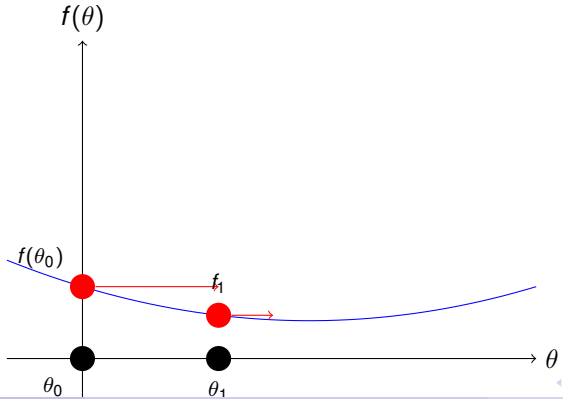

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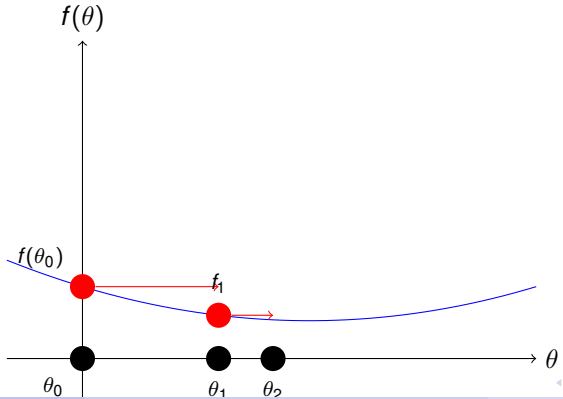

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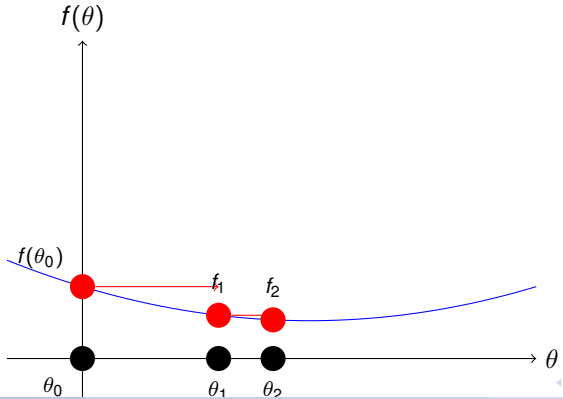

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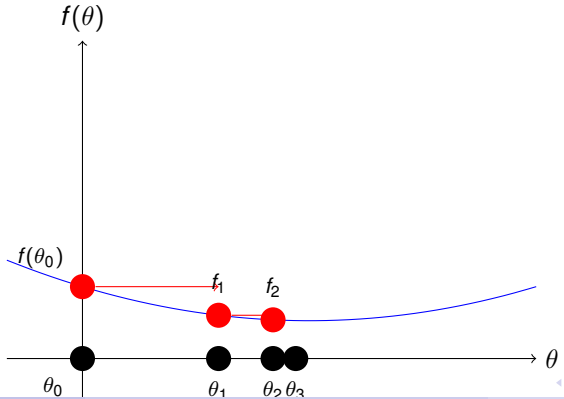

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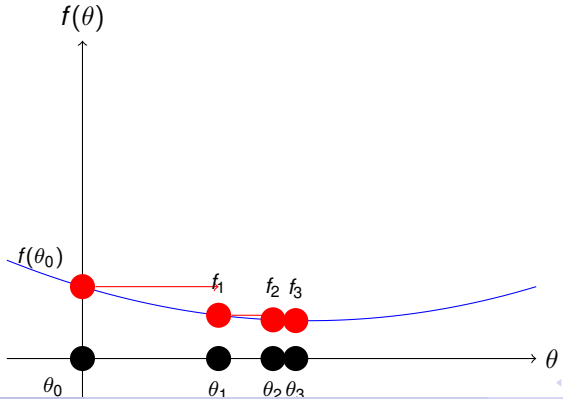

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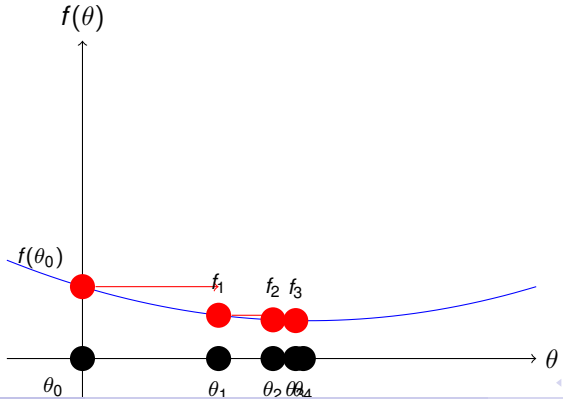

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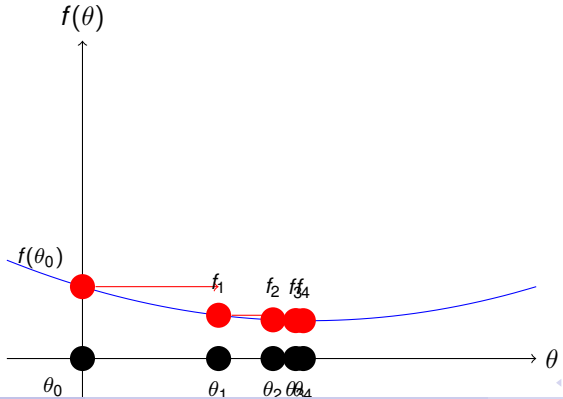

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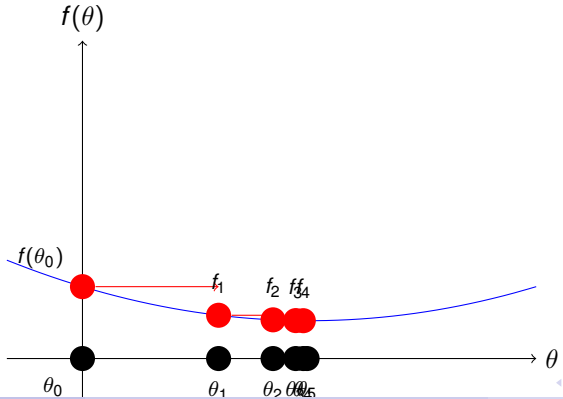

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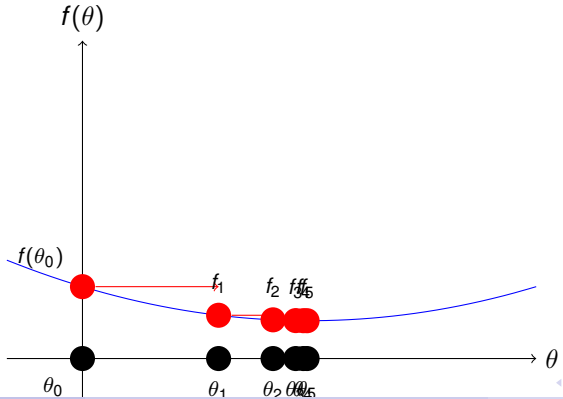

# Fonction convexe en 1D

- Le pas de gradient influe sur la convergence
- Il dépend de la fonction à optimiser
- S'il y a convergence, c'est forcément au minimum

# Difficulté 1: la haute dimension

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- Oui mais...

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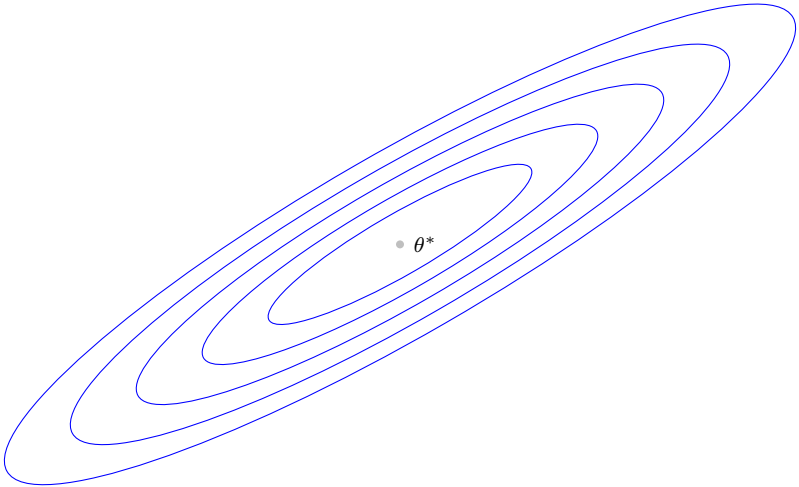

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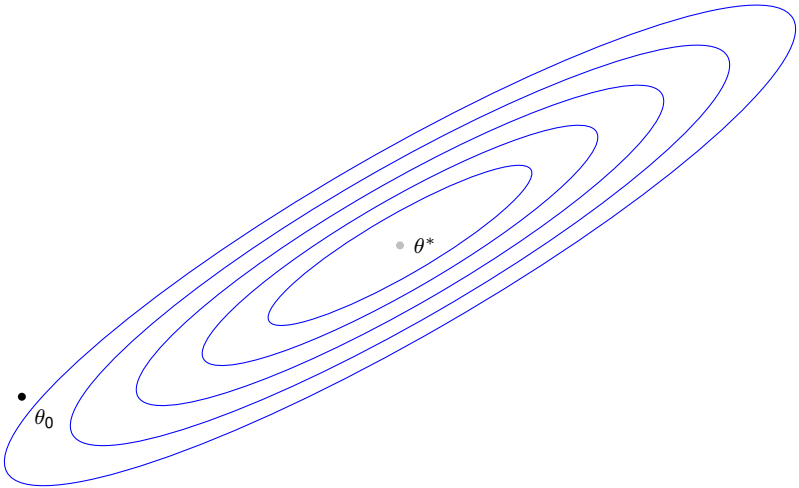

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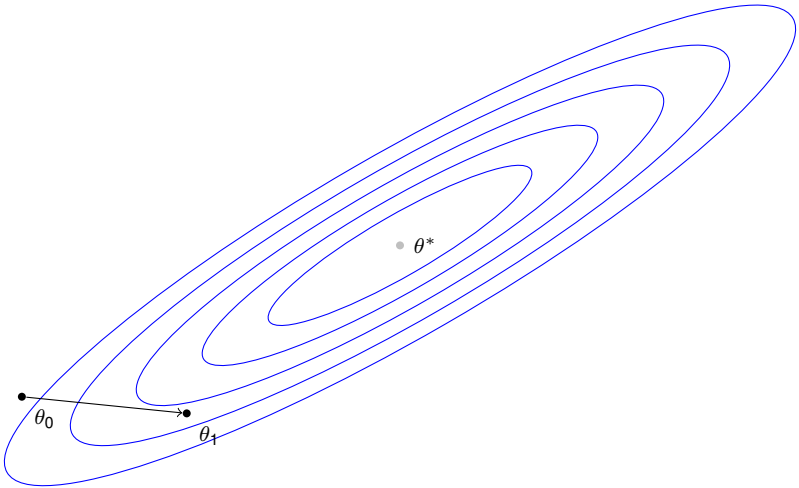

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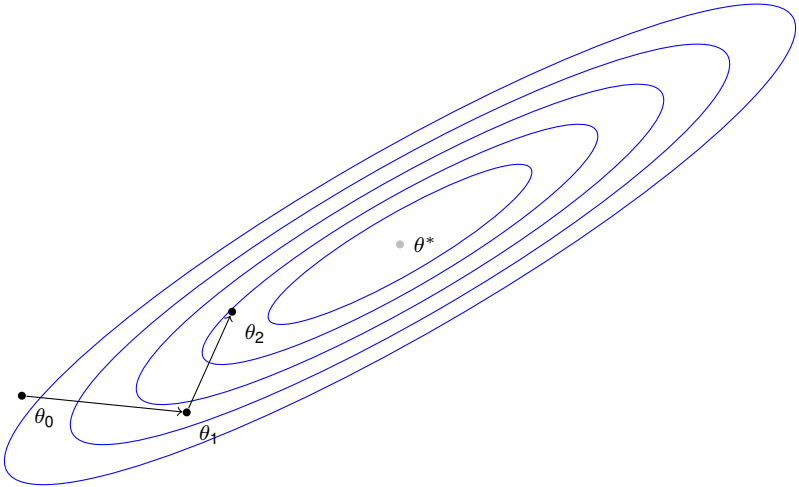

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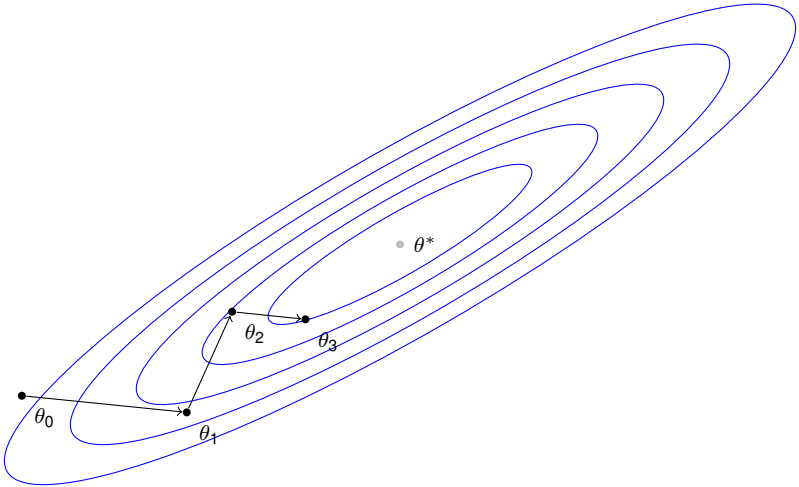

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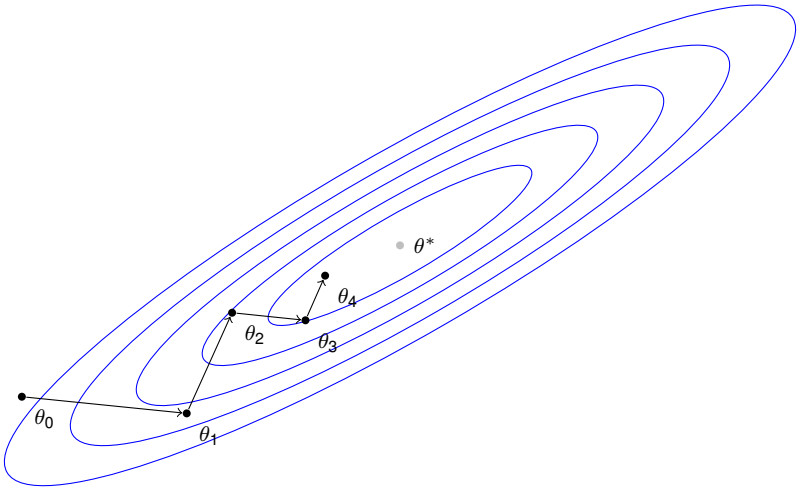

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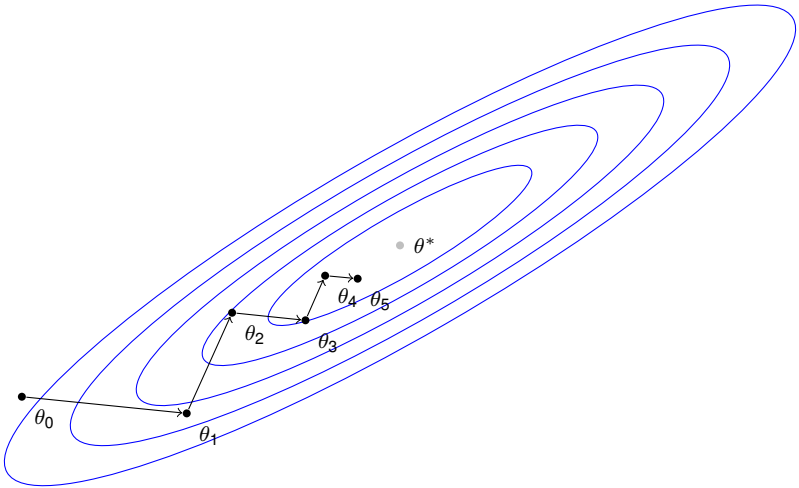

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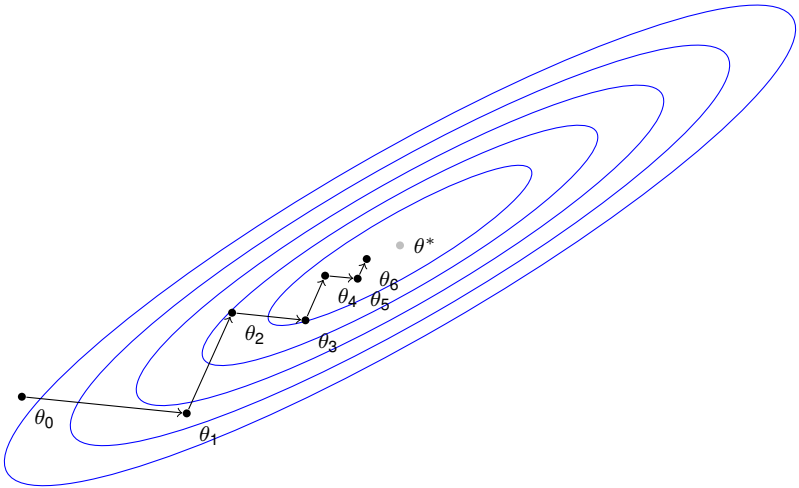

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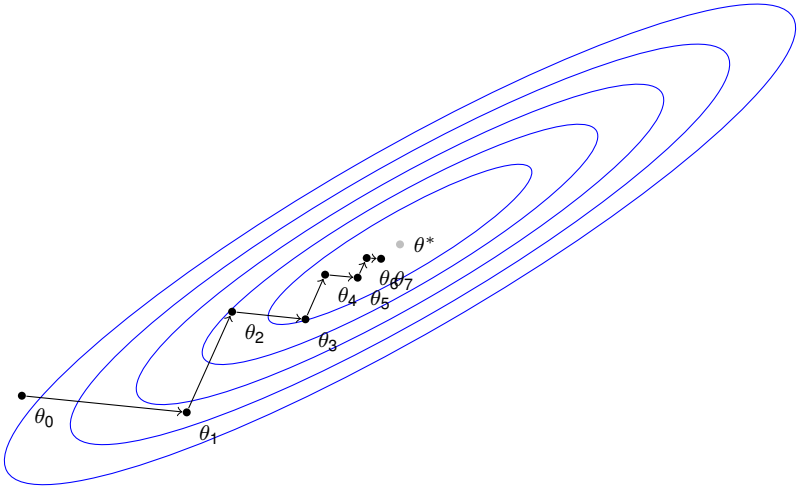

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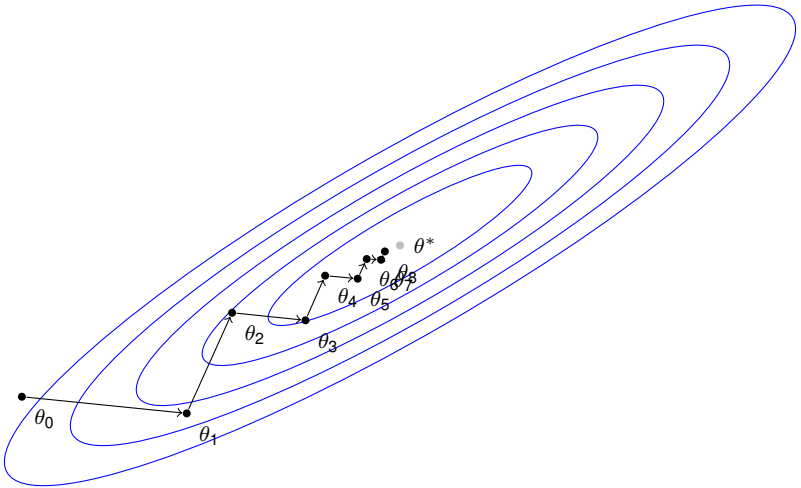

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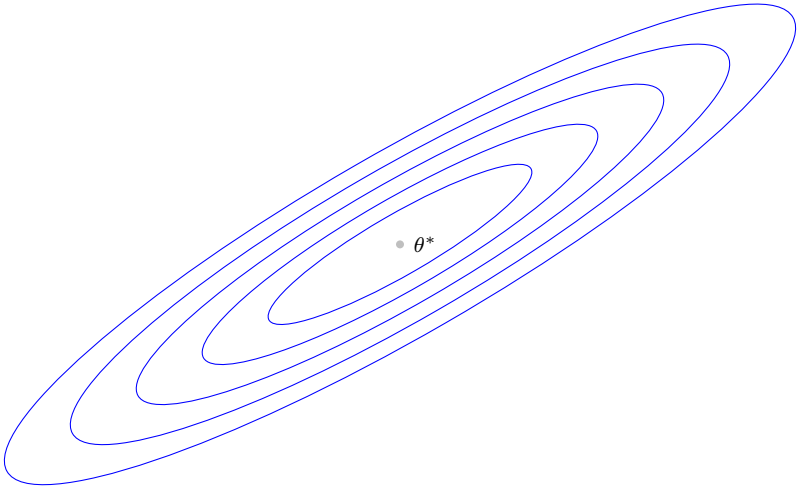

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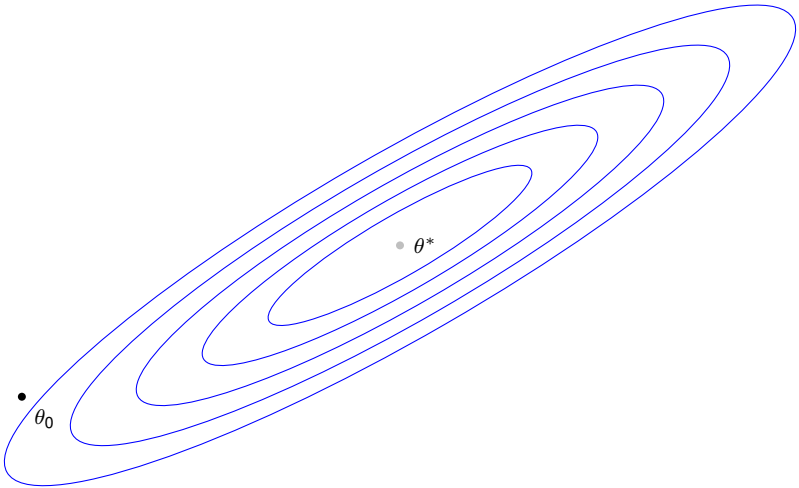

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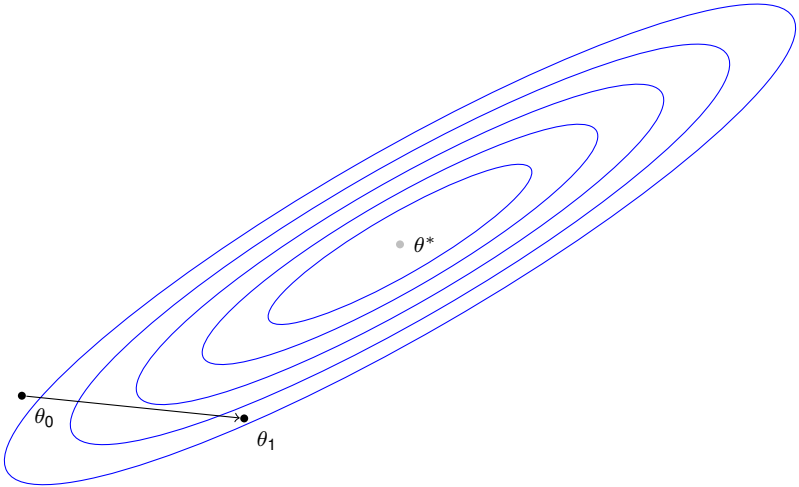

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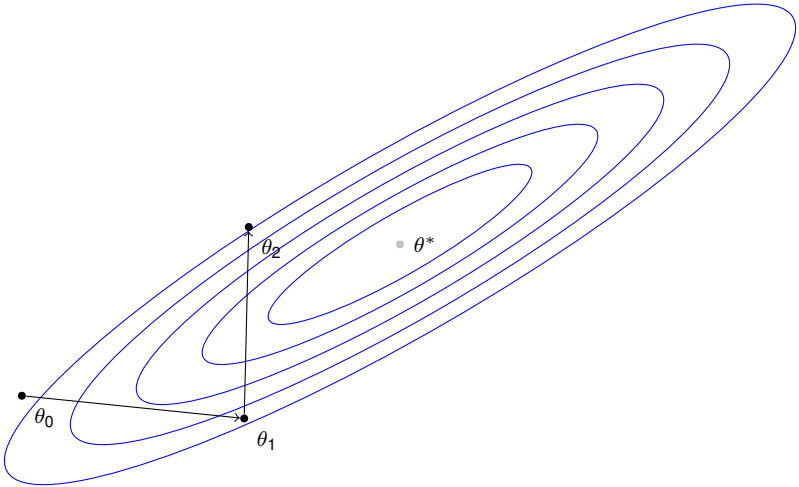

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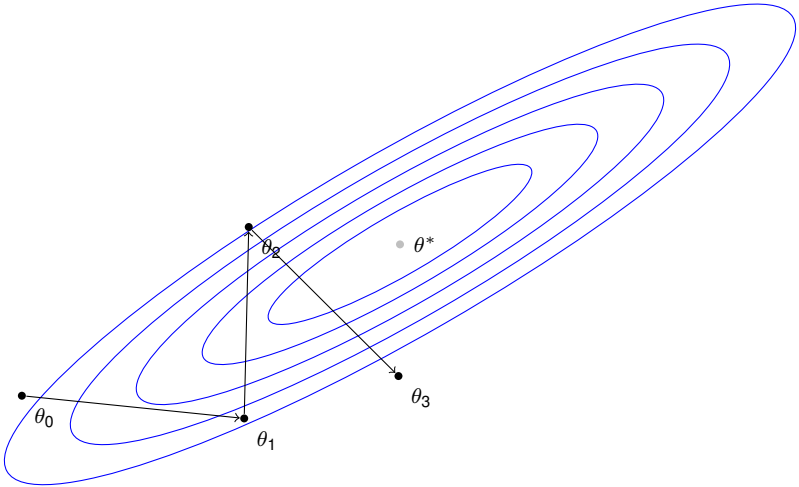

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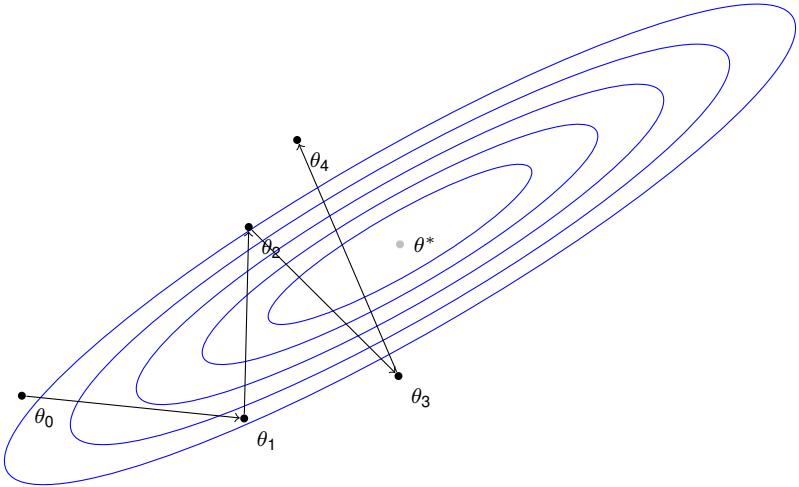

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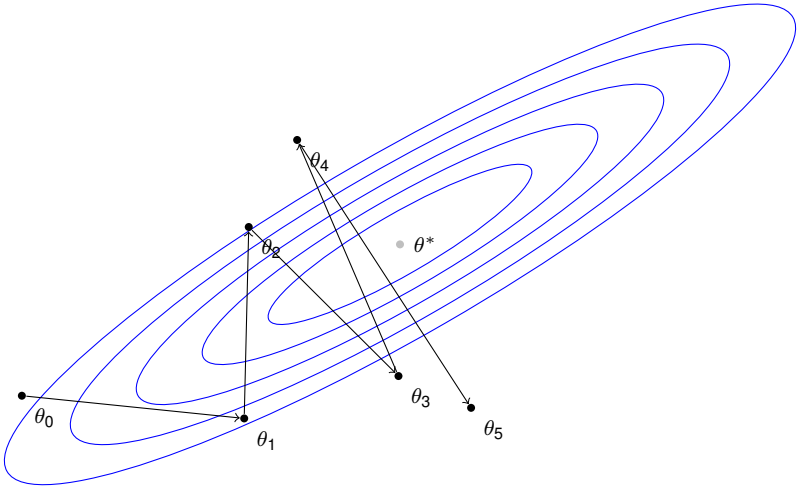

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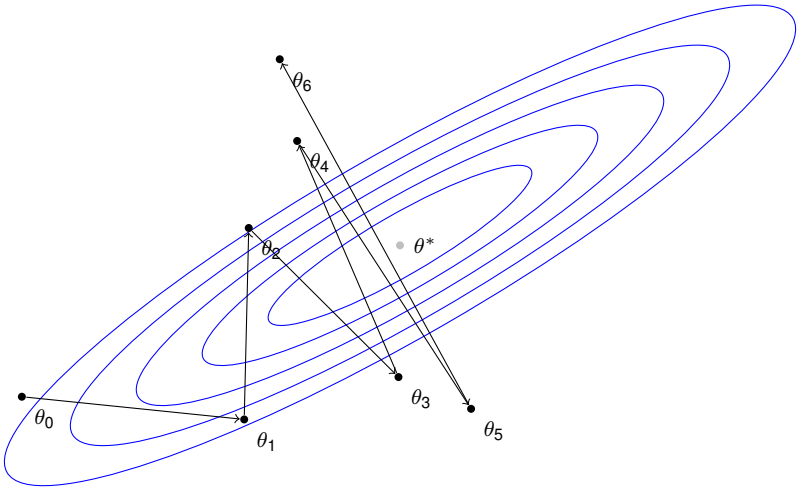

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- Certaines méthodes cherchent à *reconditionner*

# Un reconditionnement simple

- Exemple $x \in \mathbb{R}^d$
- Modèle linéaire: $f(\theta, x) = \sum_j \theta_{(j)} x_{(j)}$
- La courbure est proportionnelle à l'amplitude de $x_i$

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- Solution: normaliser les données.

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  - OUI ! [RM51]
- On a souvent $f(\theta) = \frac{1}{N} \sum_i f_i(\theta)$
- On ne va calculer le gradient que pour une seule $f_i$

# Propriétés du gradient stochastique

- L'algorithme "nécessite" un pas décroissant.
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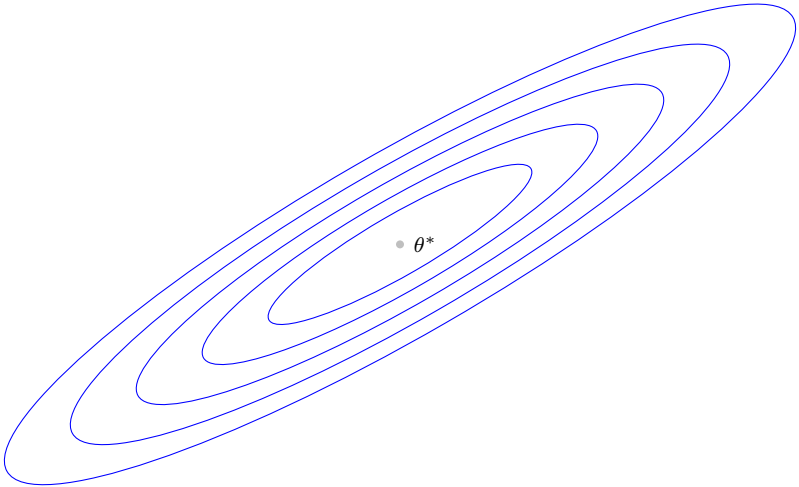
- Cela n'est vrai que pour l'erreur d'entraînement.
- Même convergence sur l'erreur de test [BB08].
  - Convergence en nombre de *mises à jour*.

# Gradient stochastique - Pas constant



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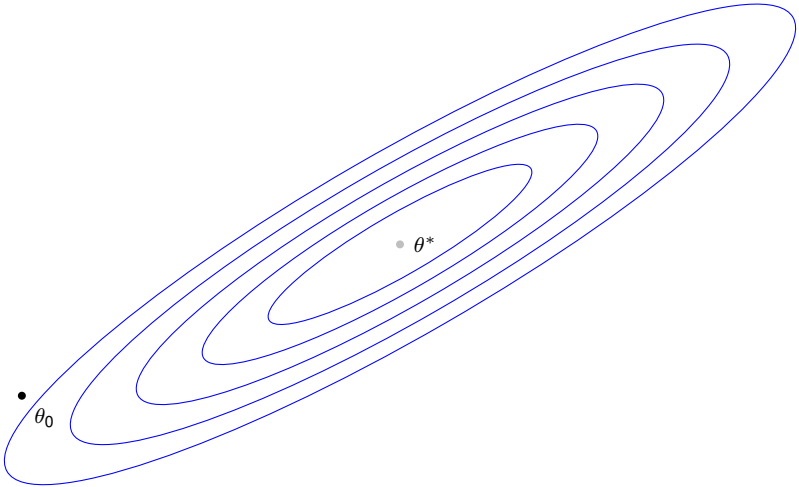

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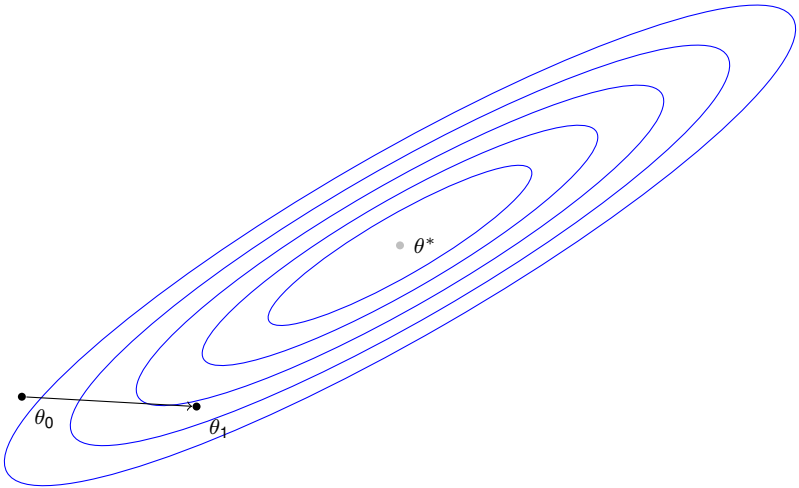

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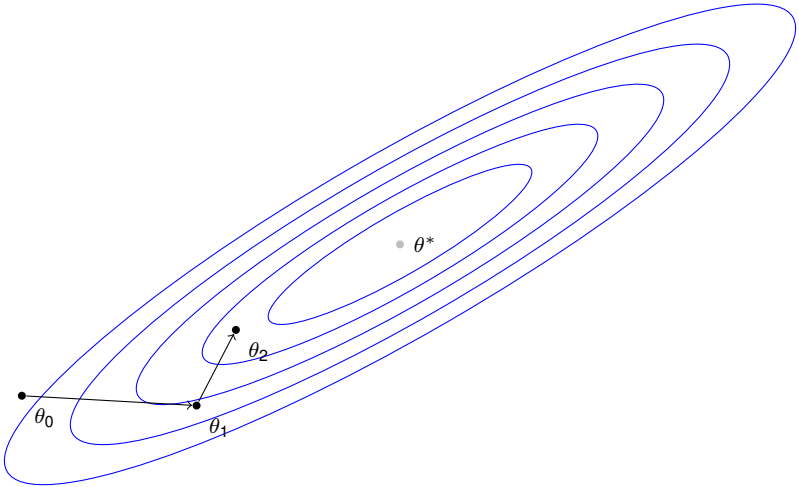

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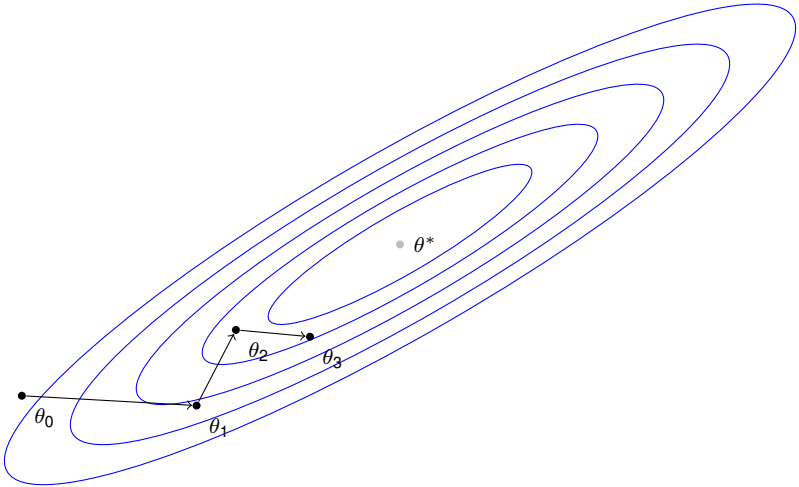

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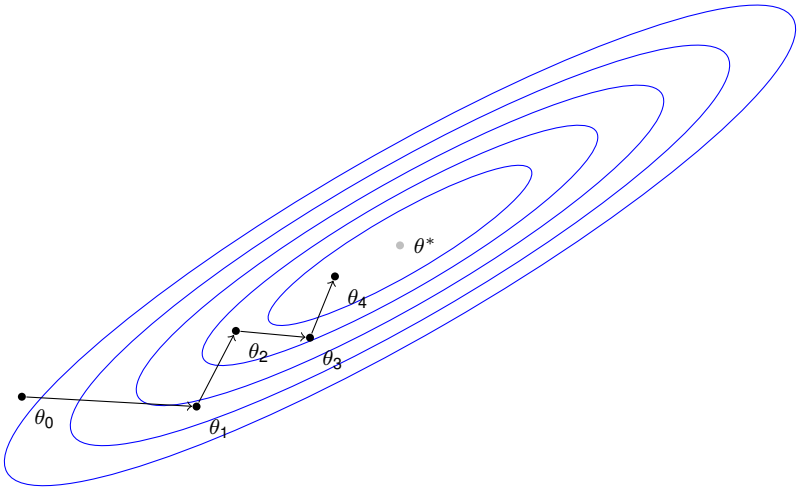

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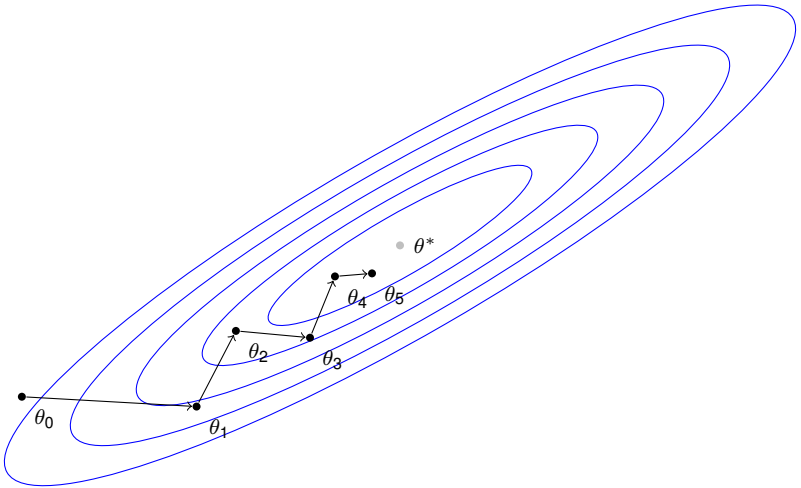

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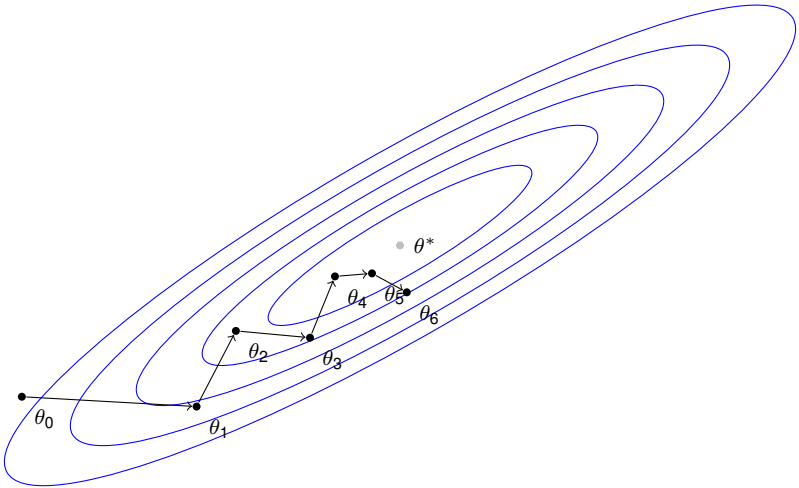

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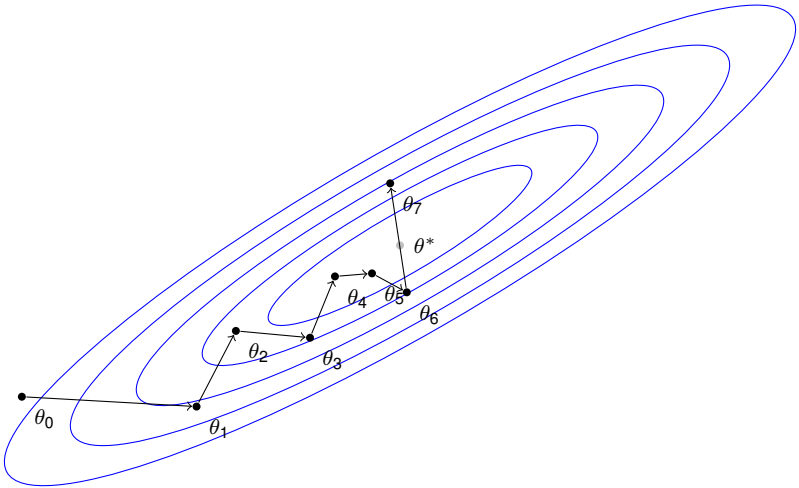

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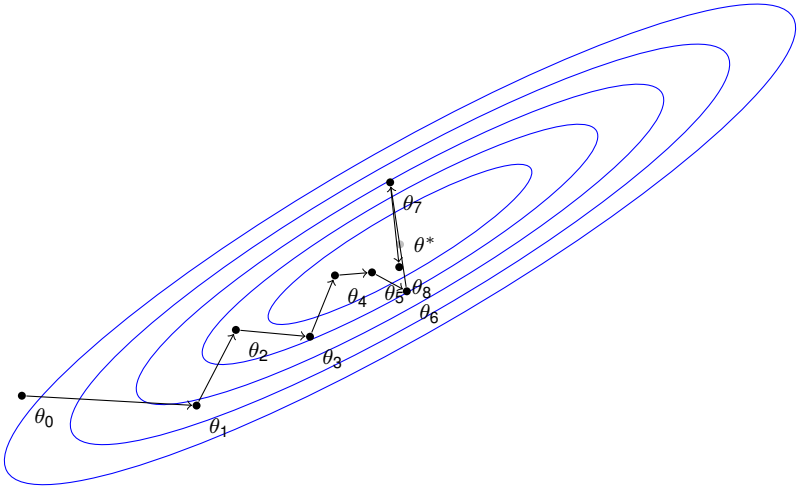

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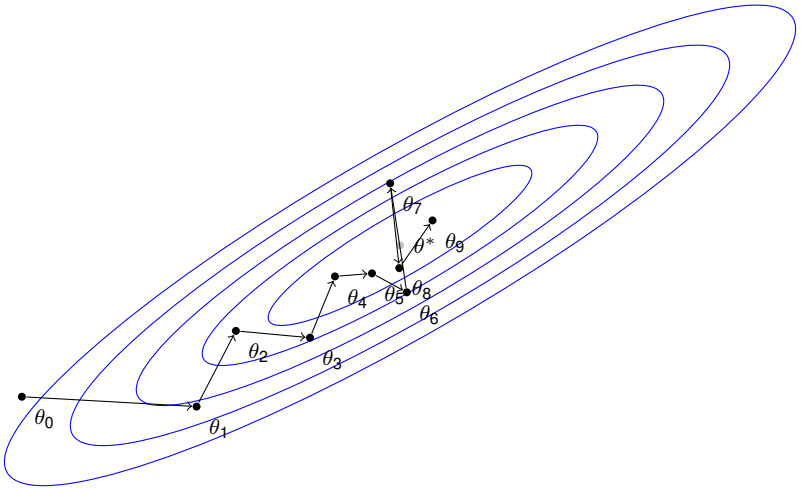

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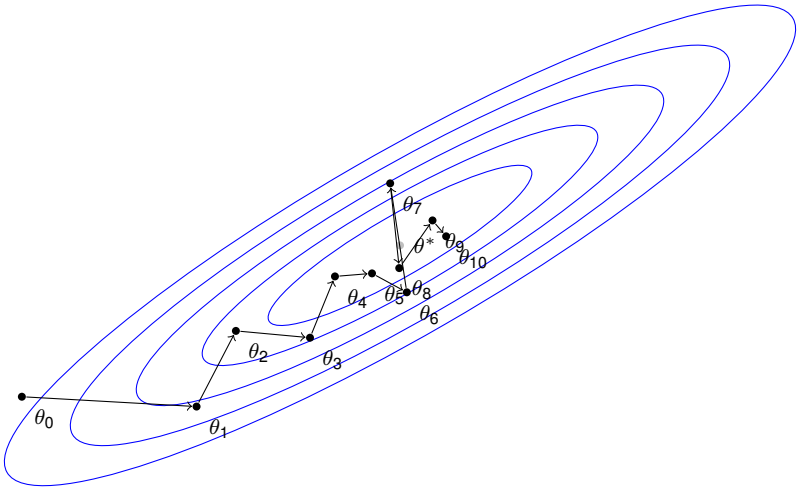

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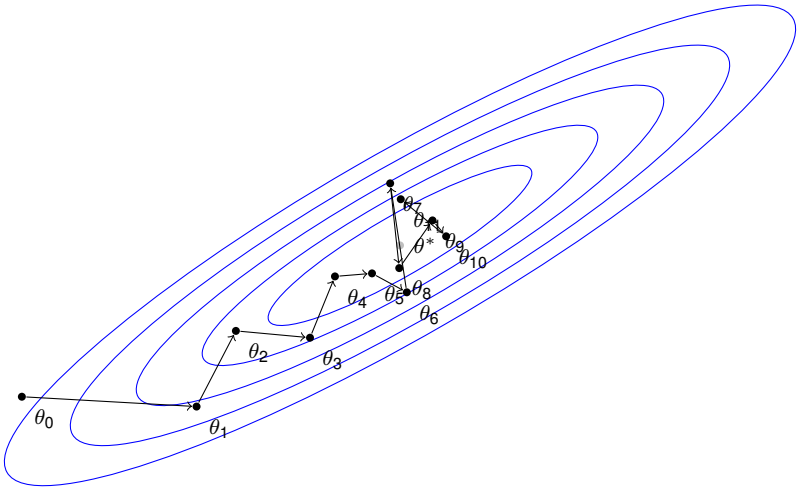

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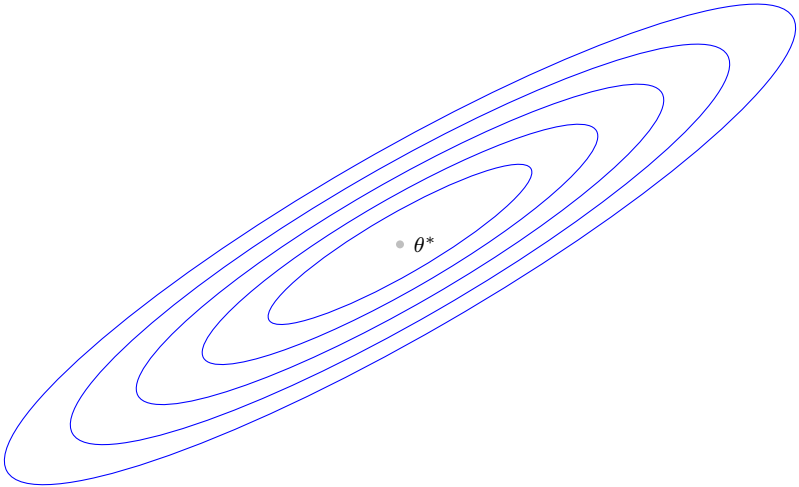

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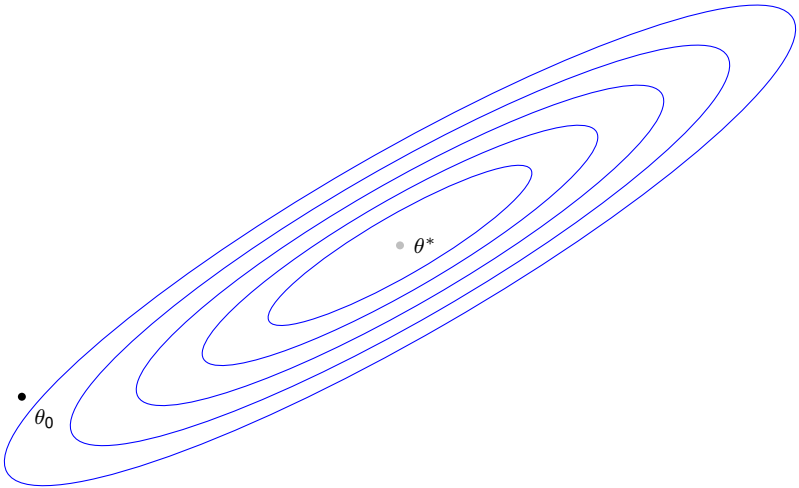

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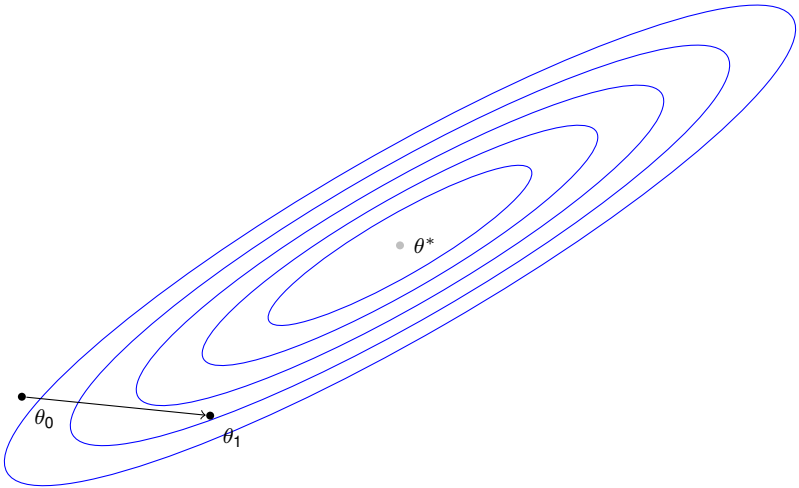

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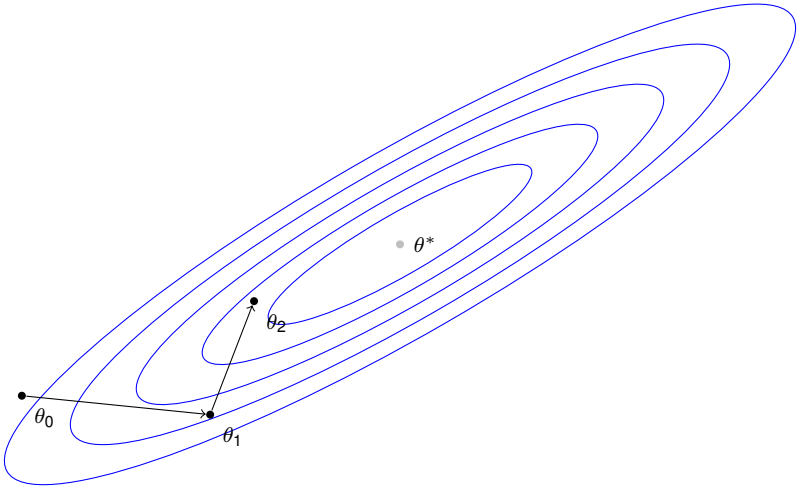

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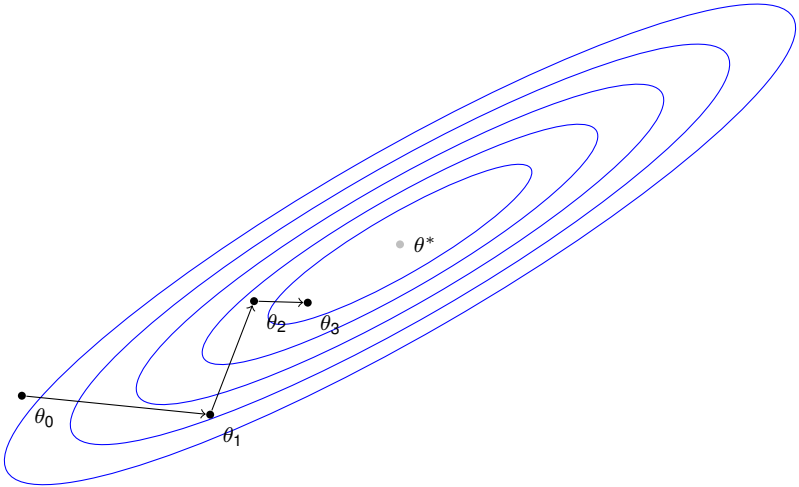

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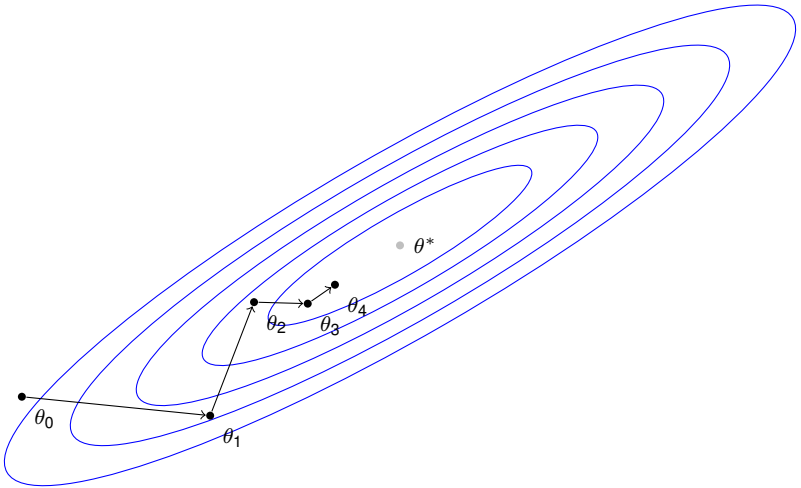

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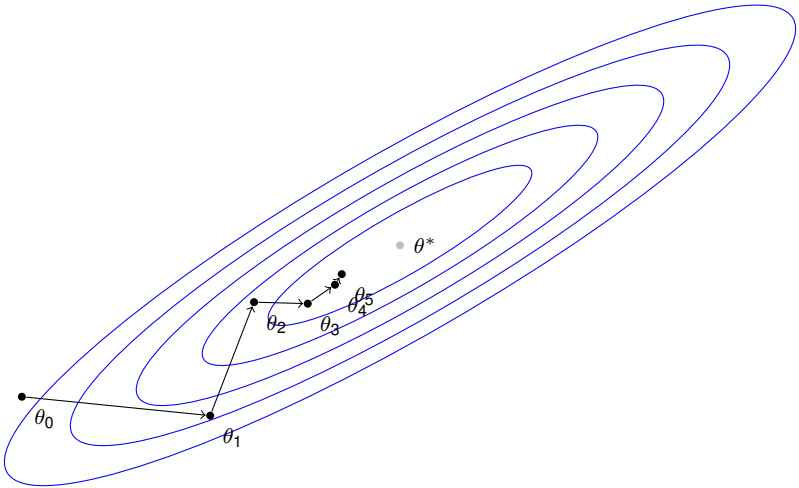

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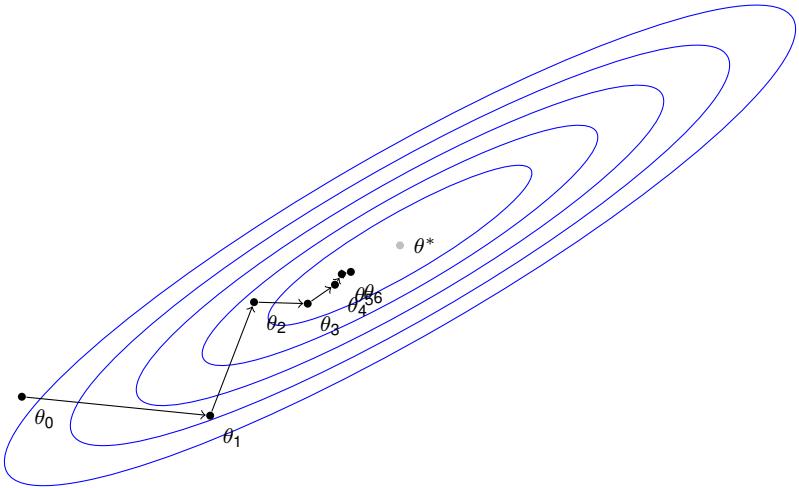

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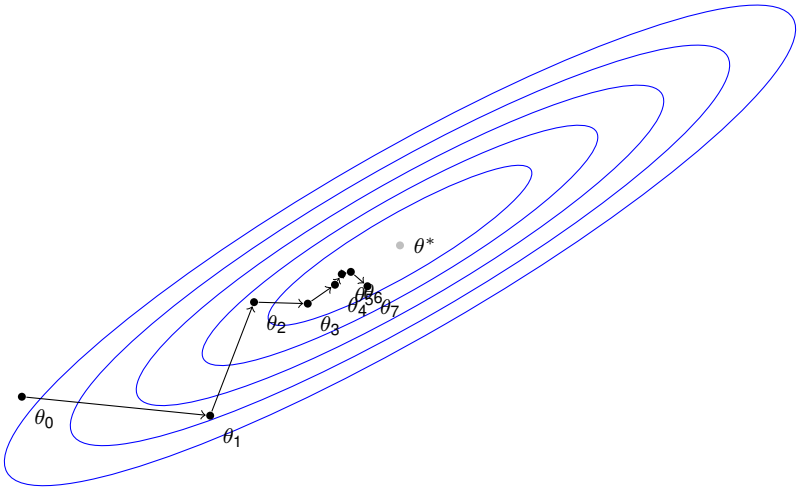

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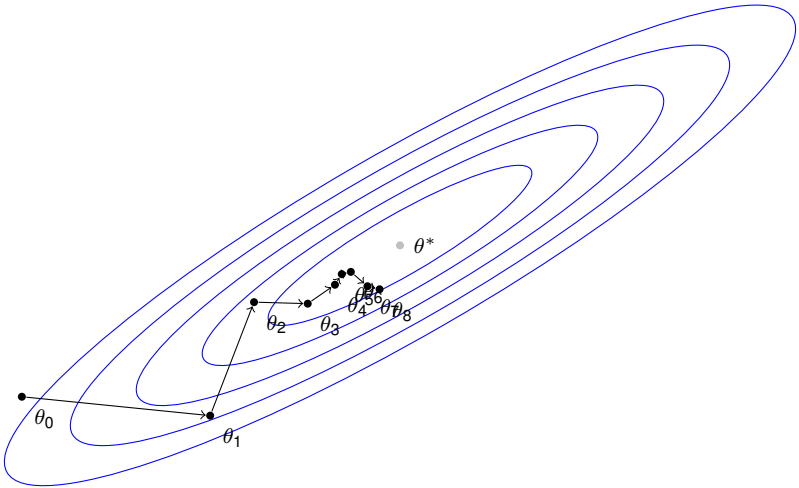

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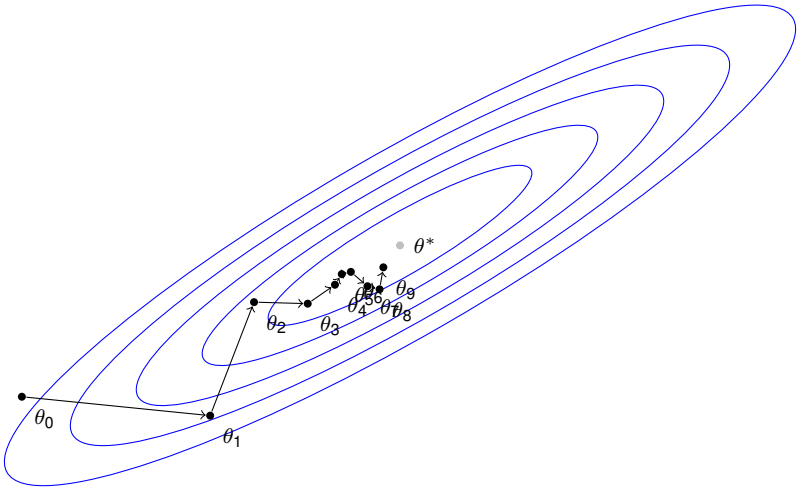

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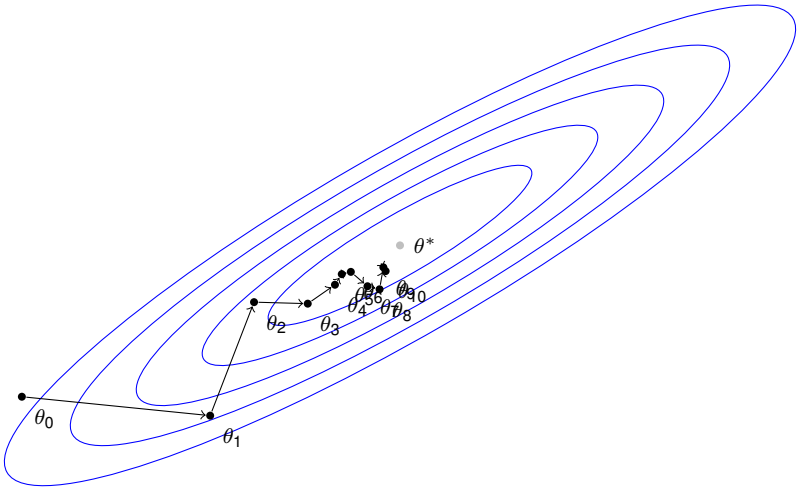

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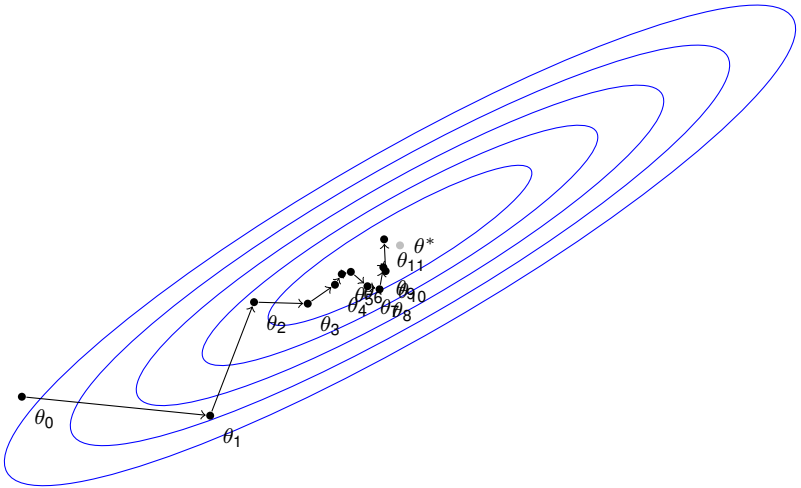

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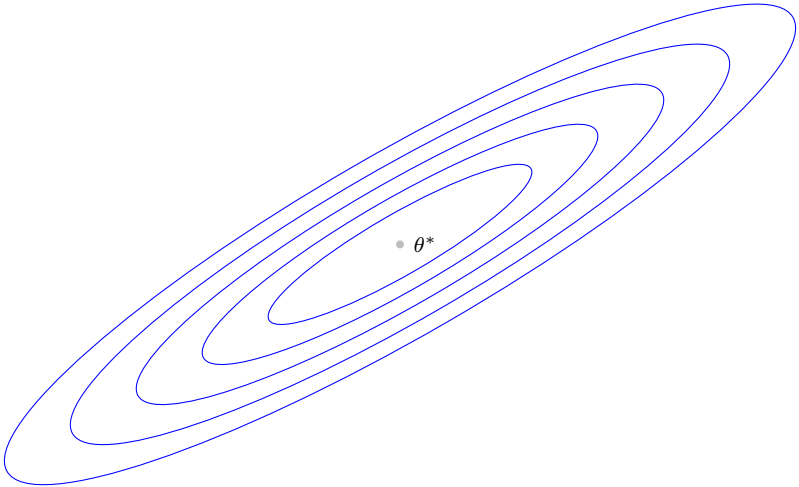

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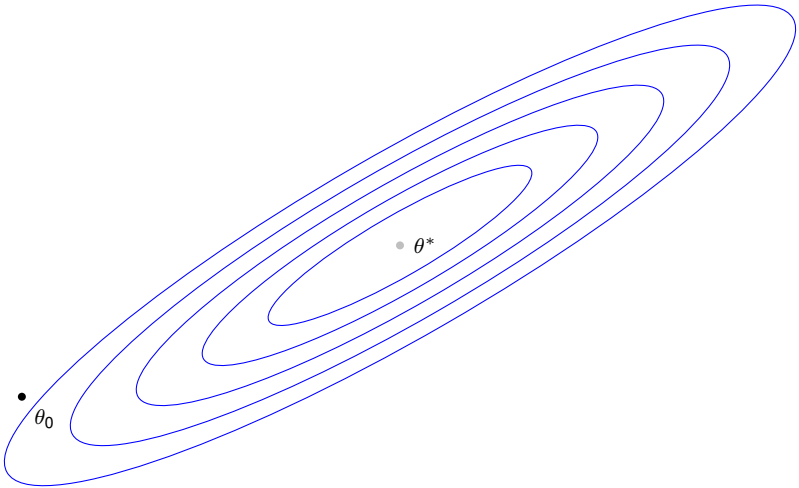

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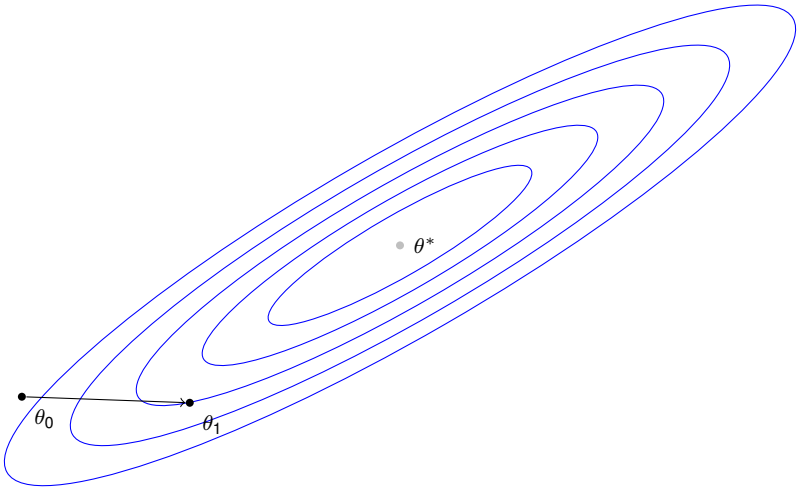

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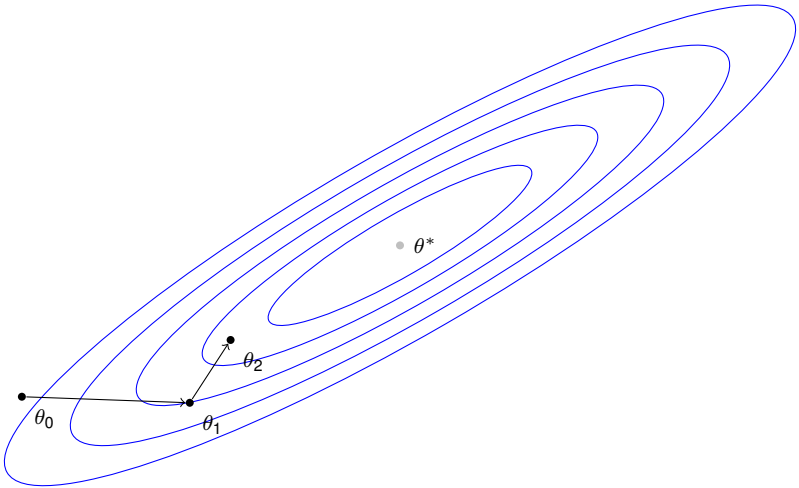

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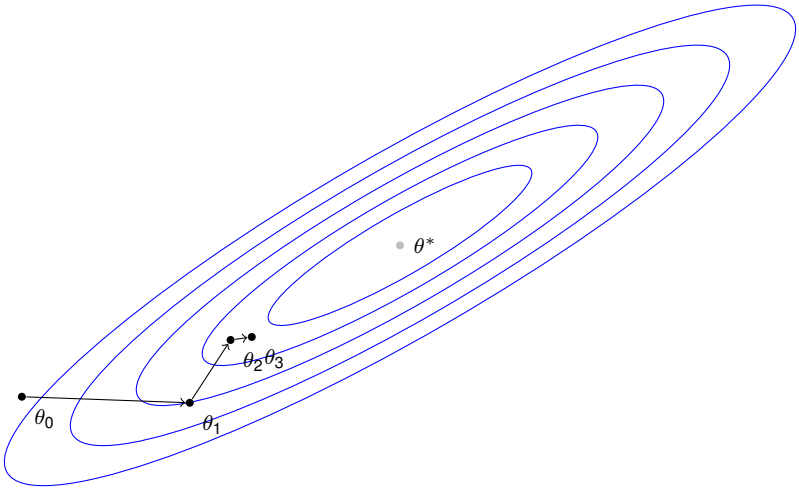

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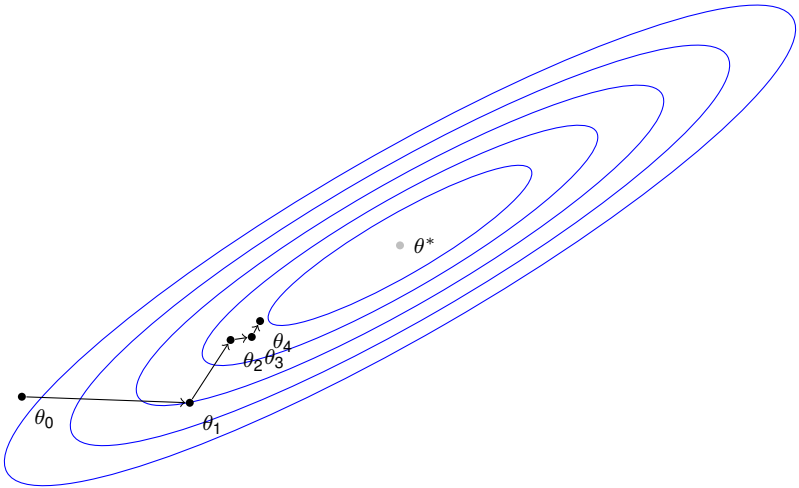

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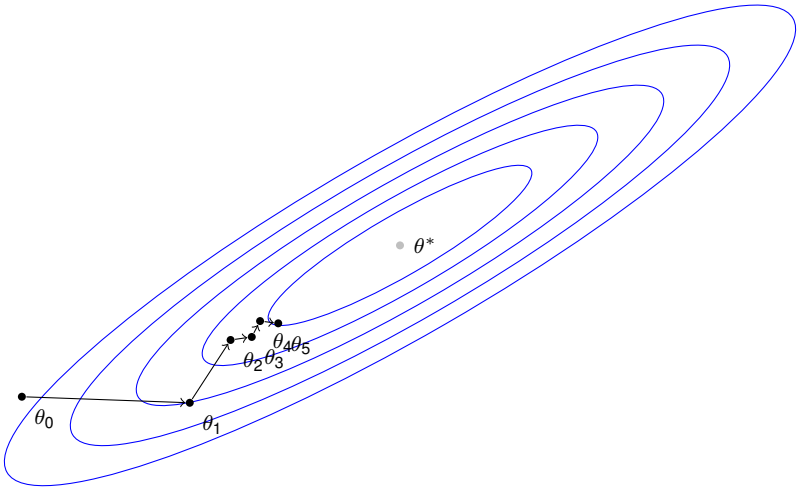

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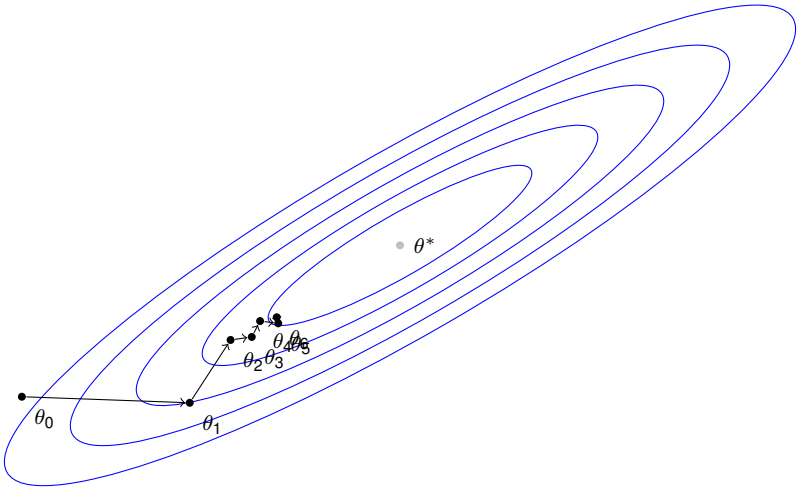

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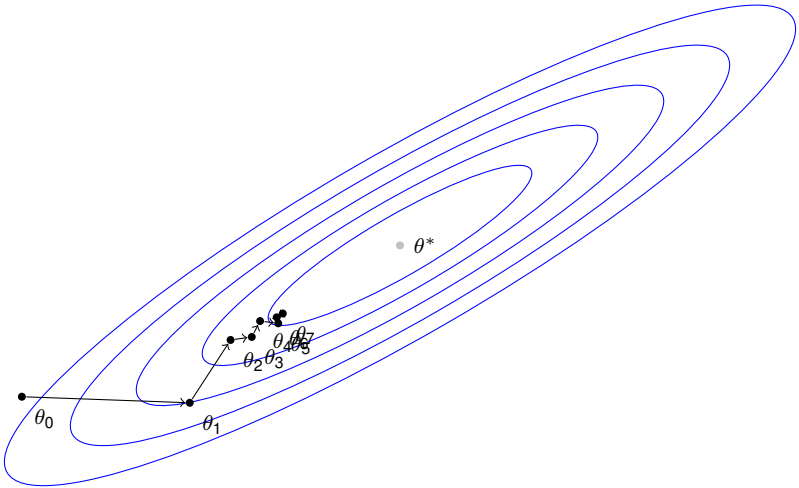

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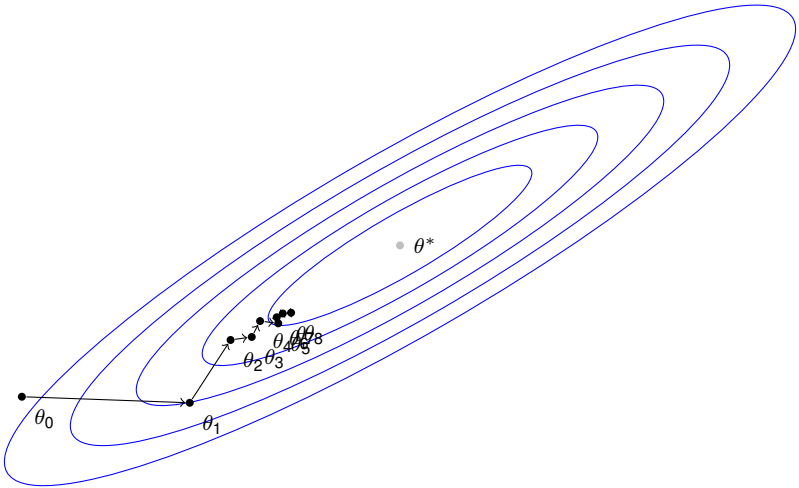

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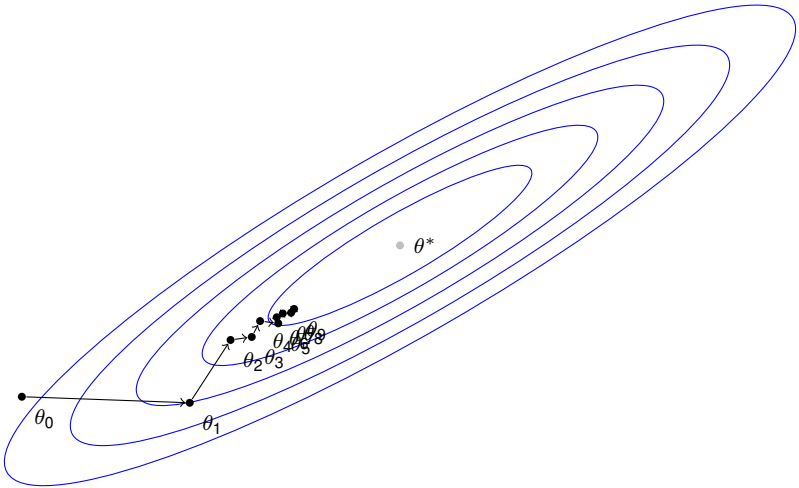

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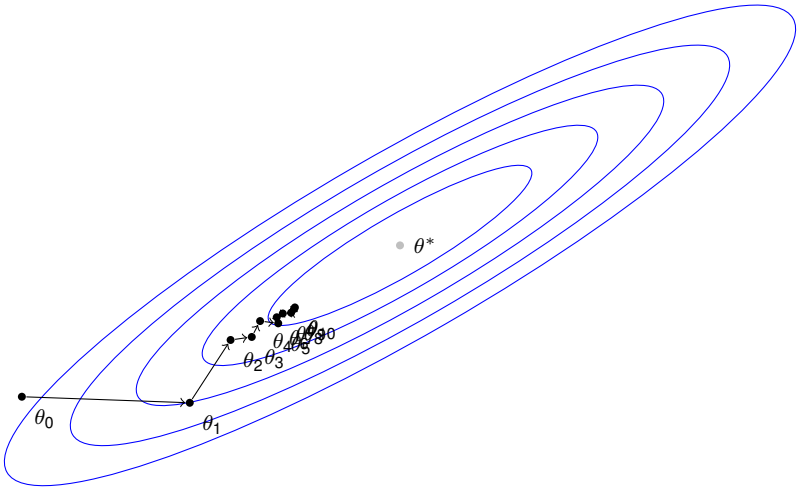

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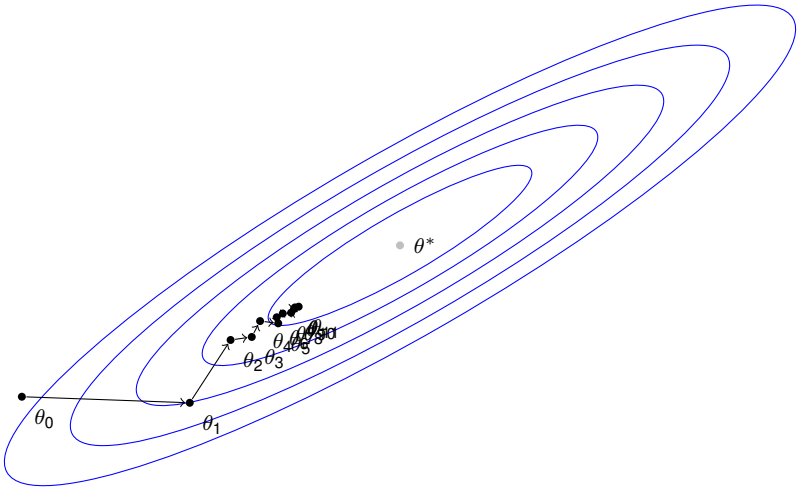

# Difficulté 3: non-convexité

- Preuves de convergence valides dans le cas convexe
- Les réseaux profonds sont non-convexes
- Théorie encore balbutiante mais des résultats empiriques

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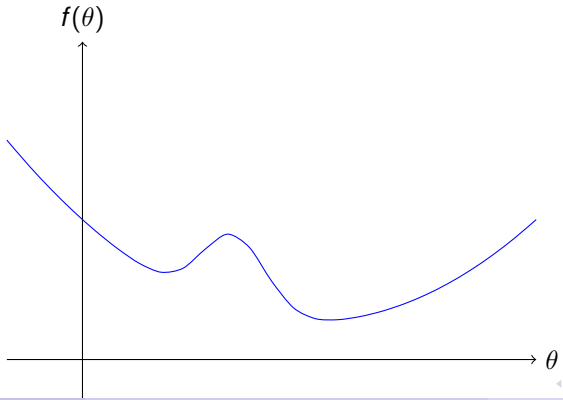

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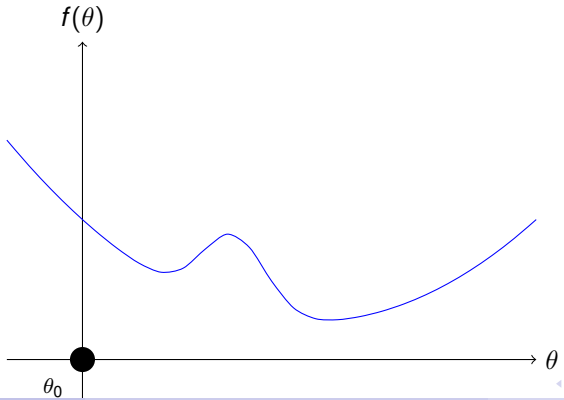

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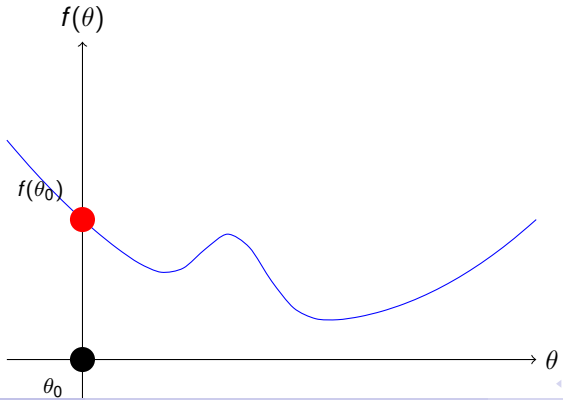

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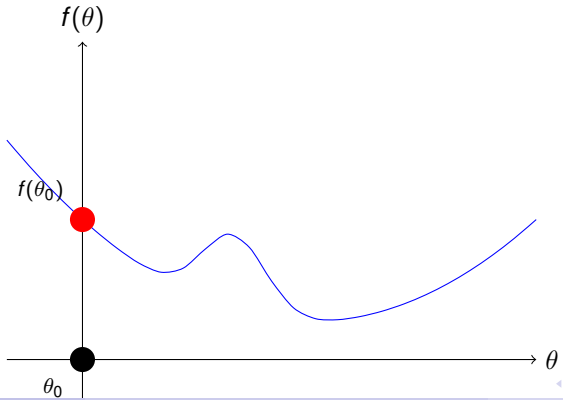

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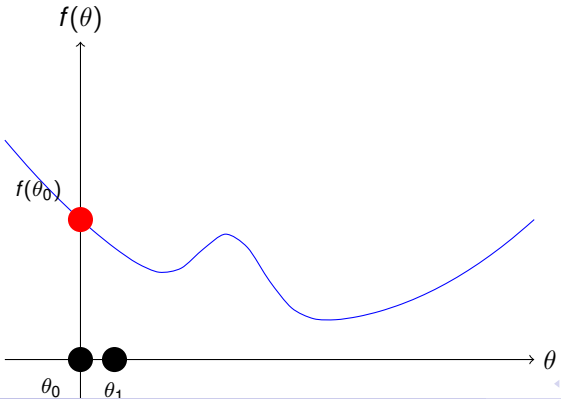

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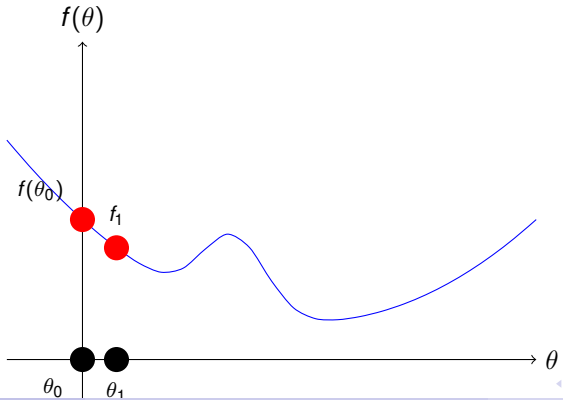

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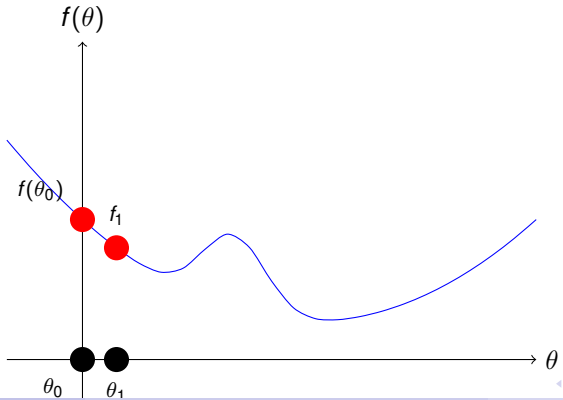

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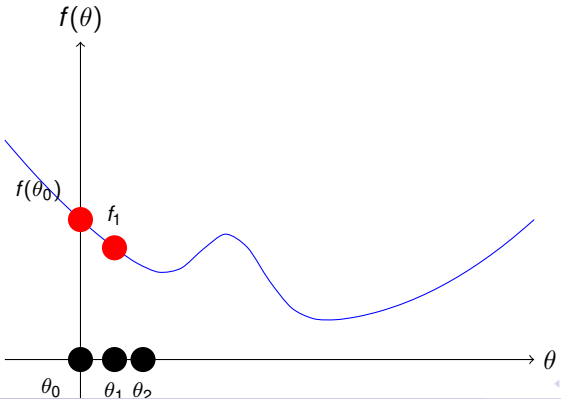

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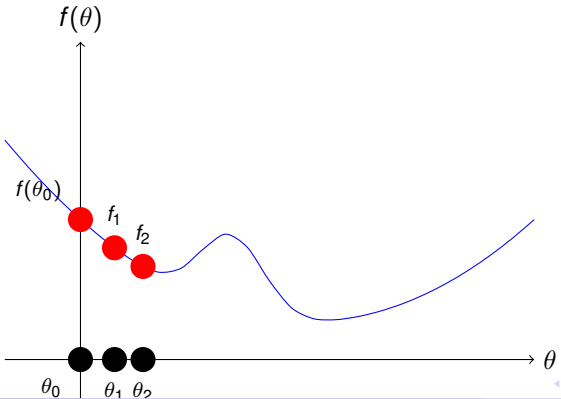

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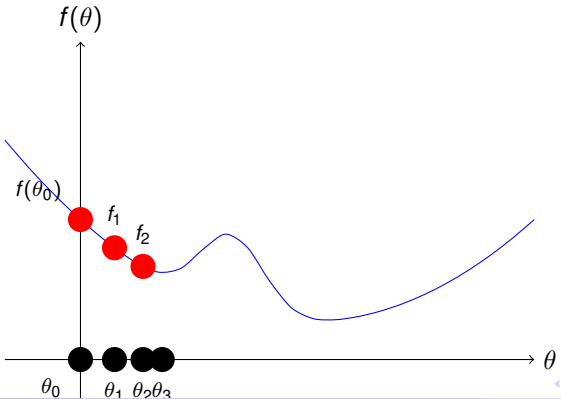

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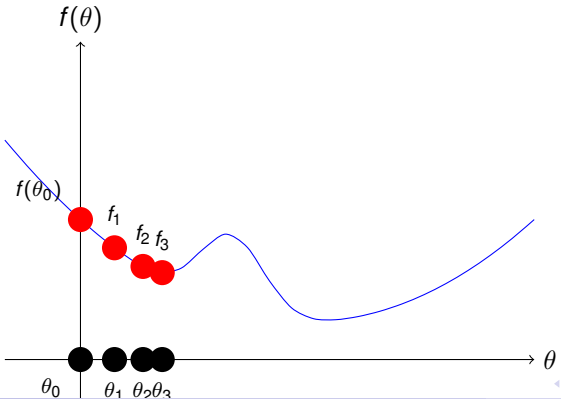

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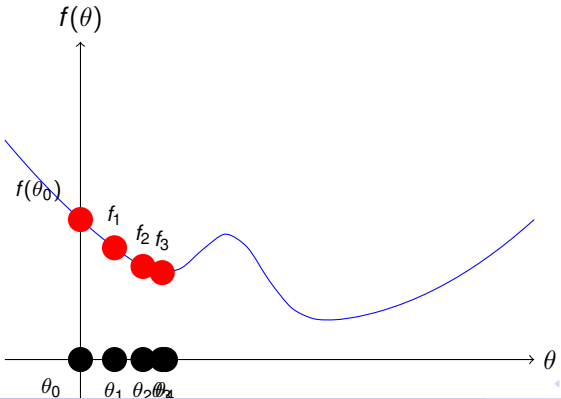

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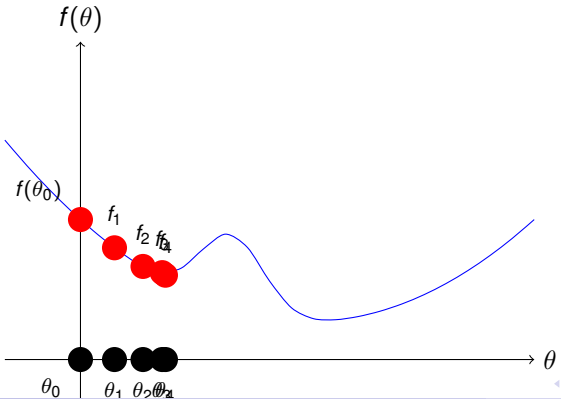

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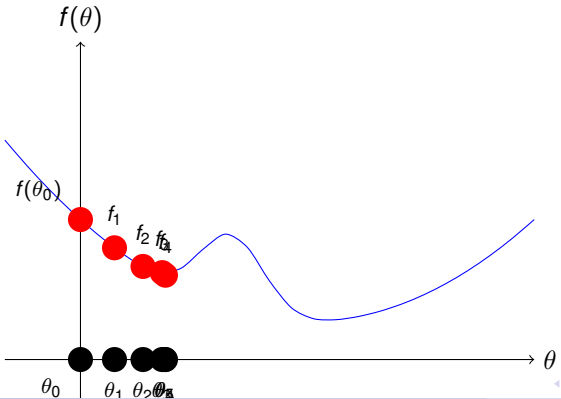

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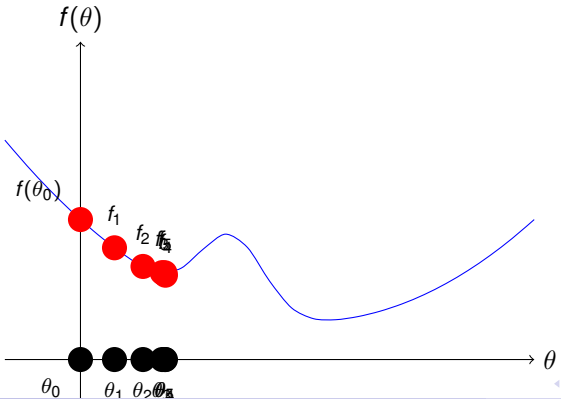

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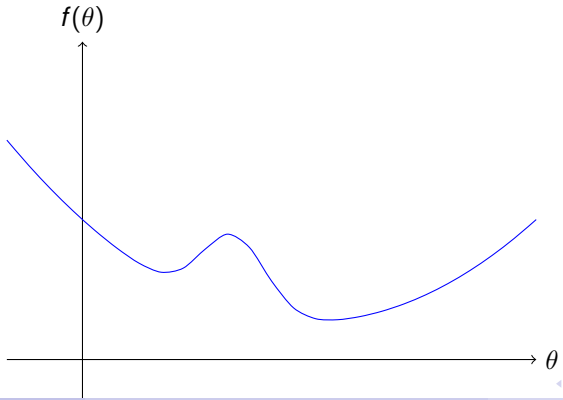

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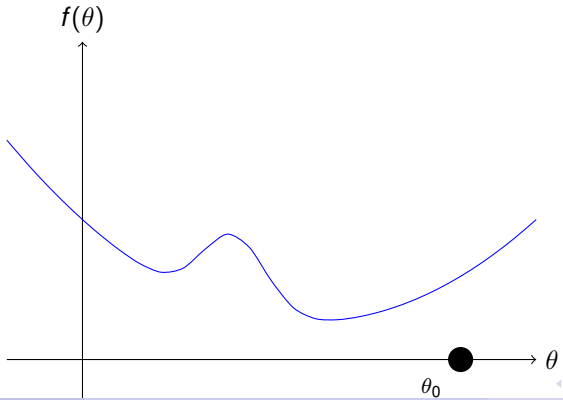

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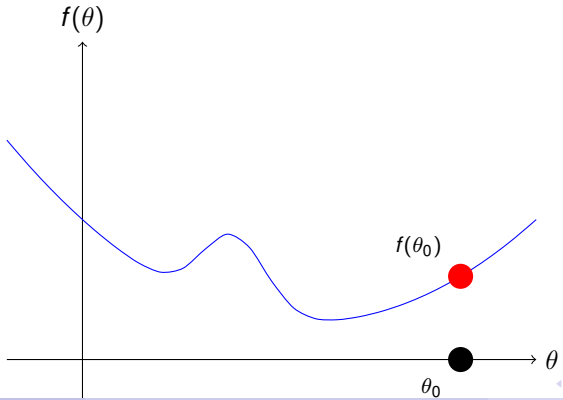

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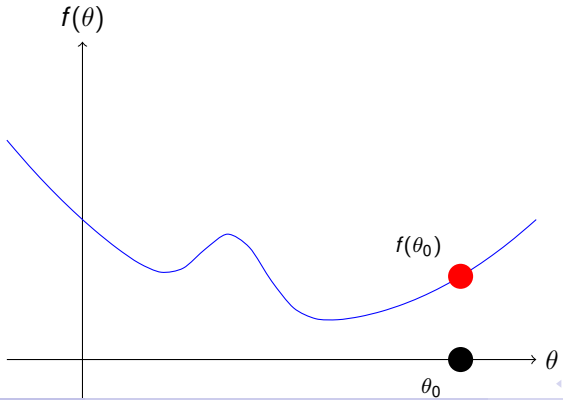

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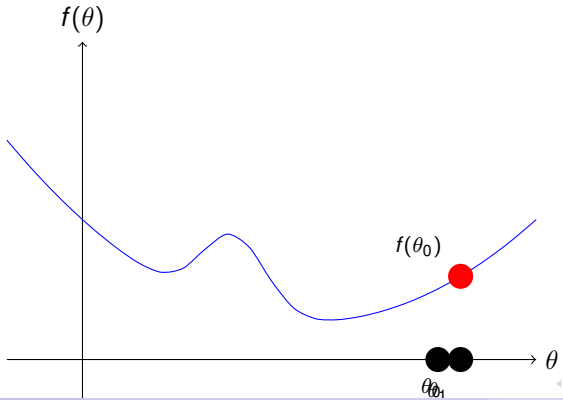

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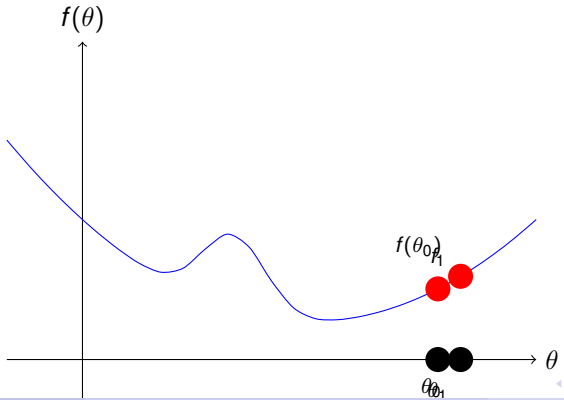

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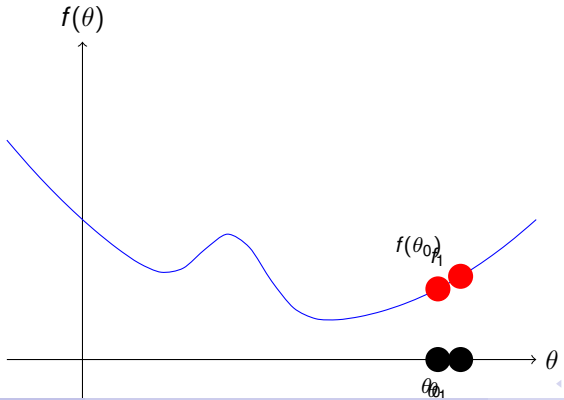

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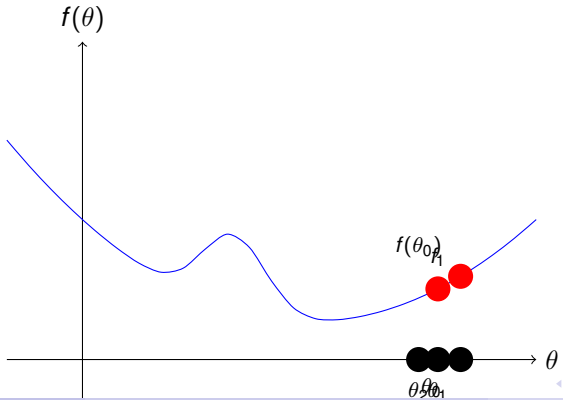

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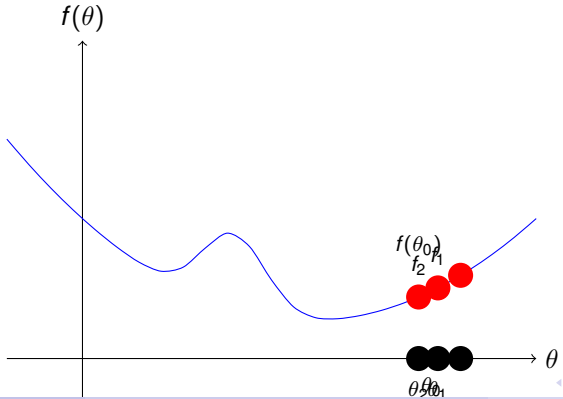

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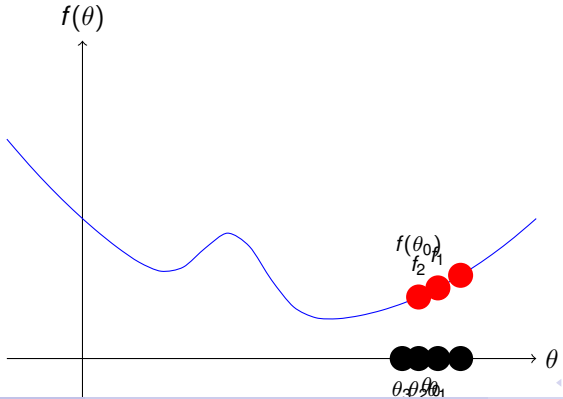

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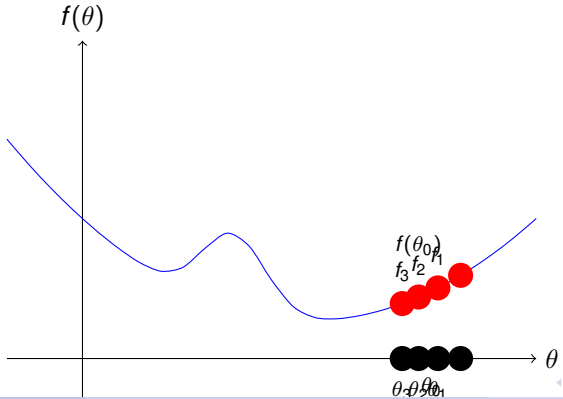

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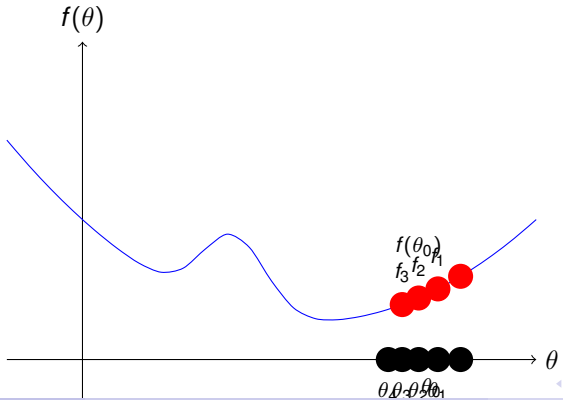

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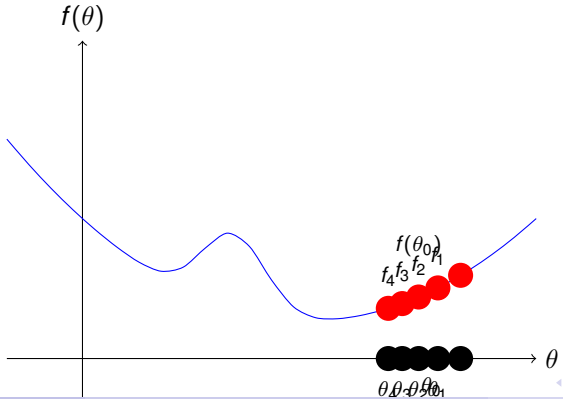

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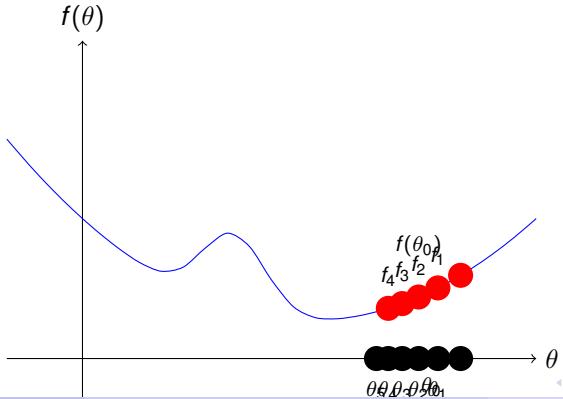

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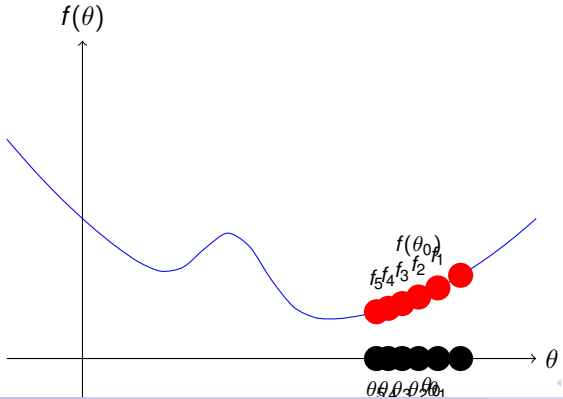

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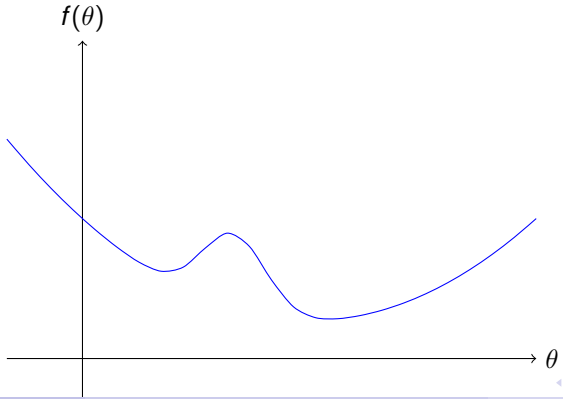

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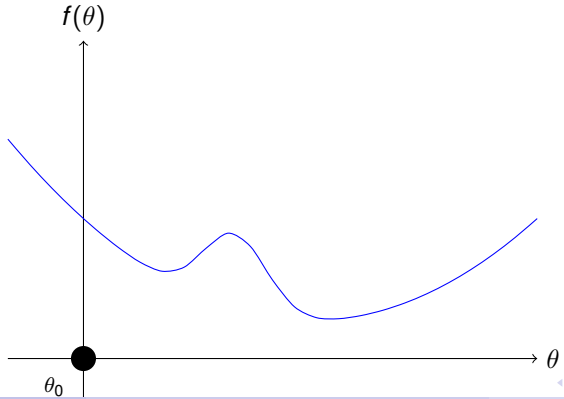

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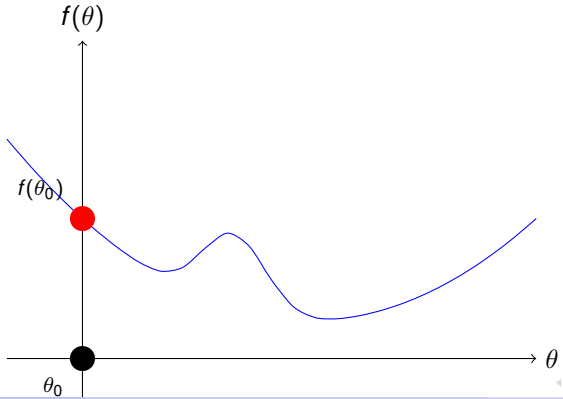

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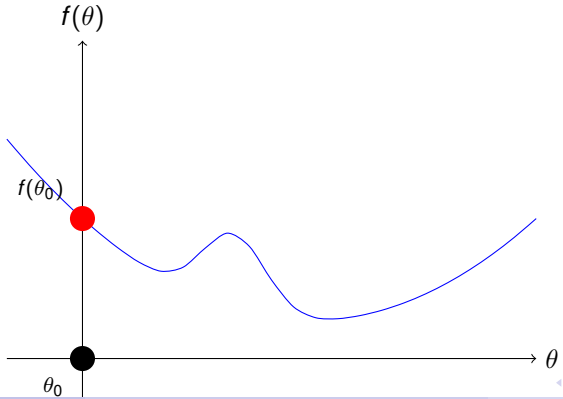

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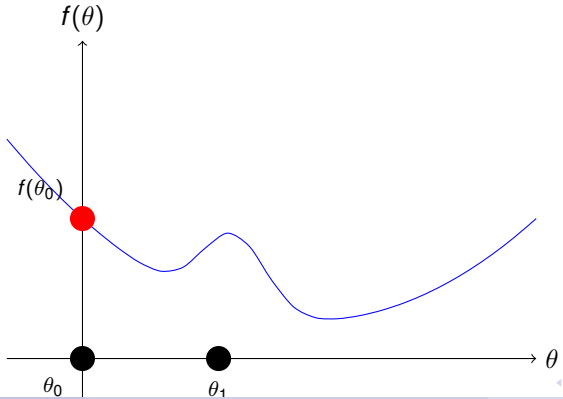

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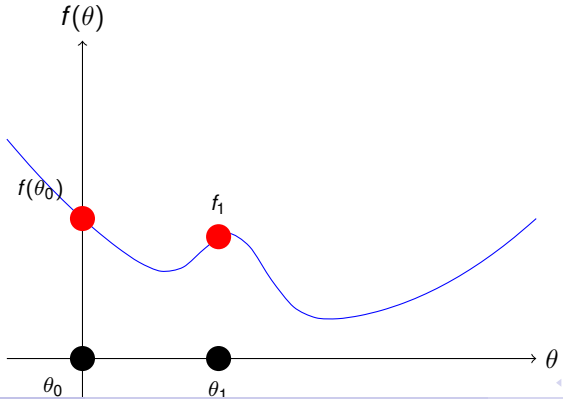

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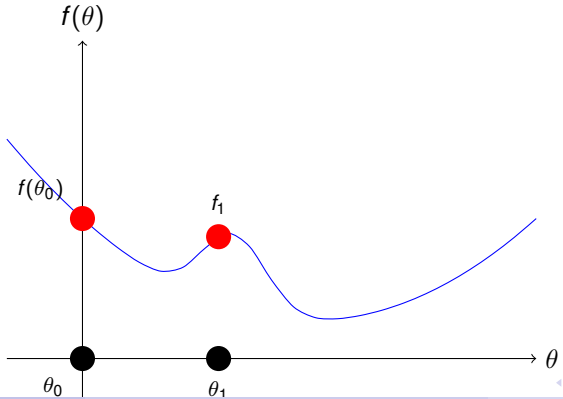

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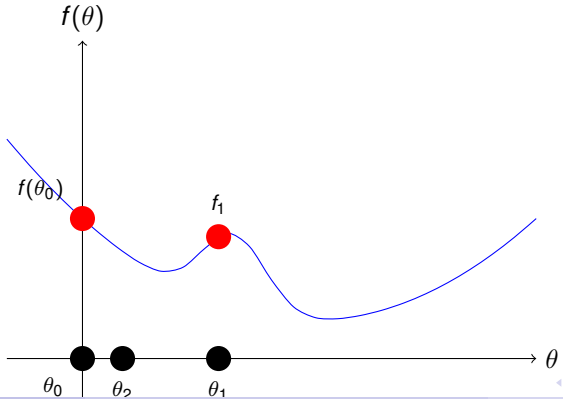

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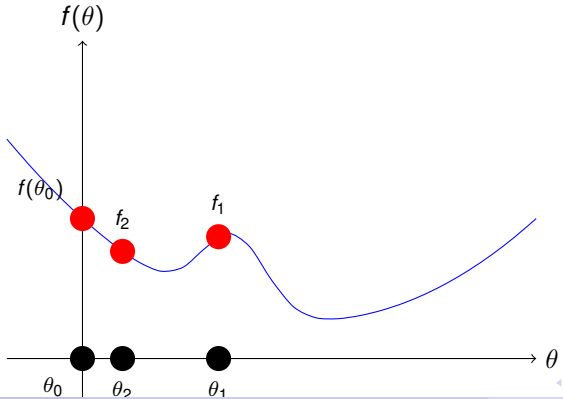

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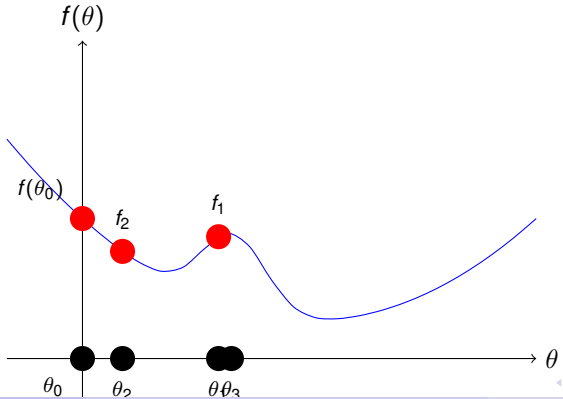

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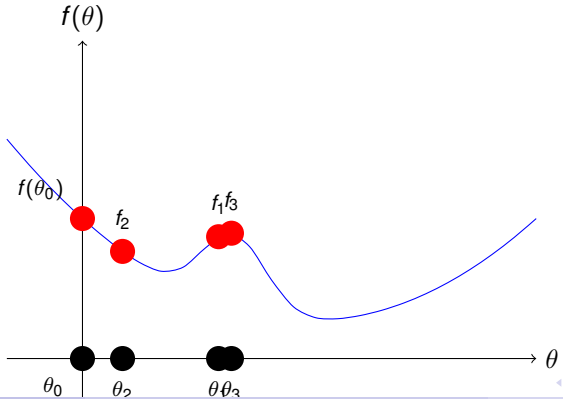

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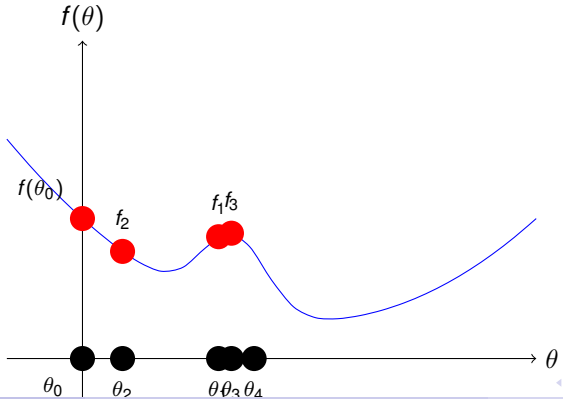

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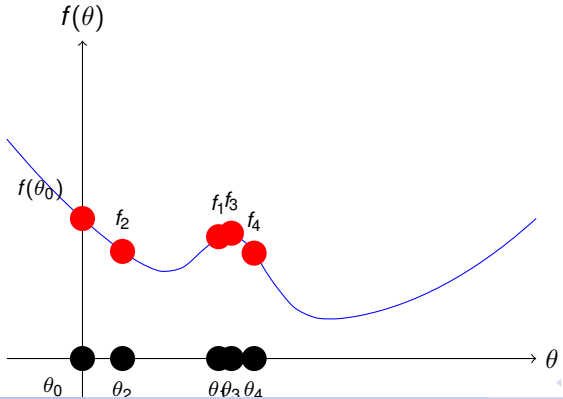

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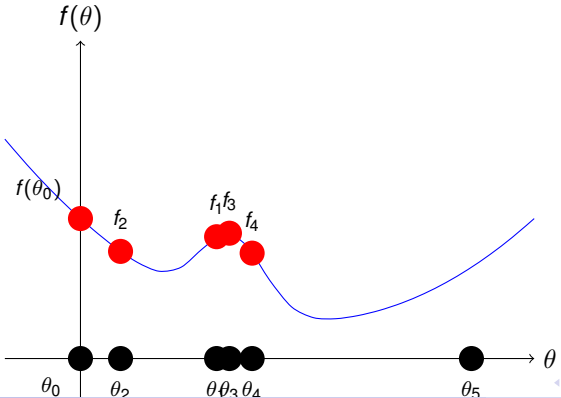

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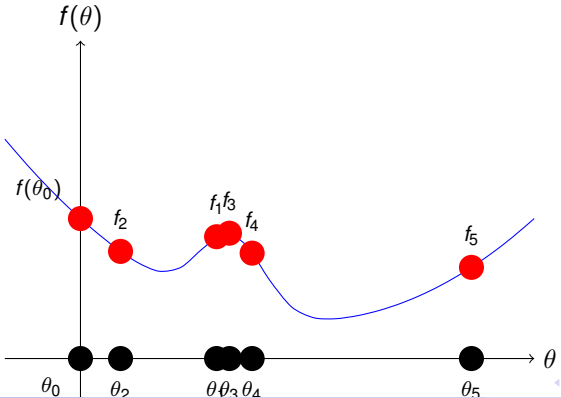

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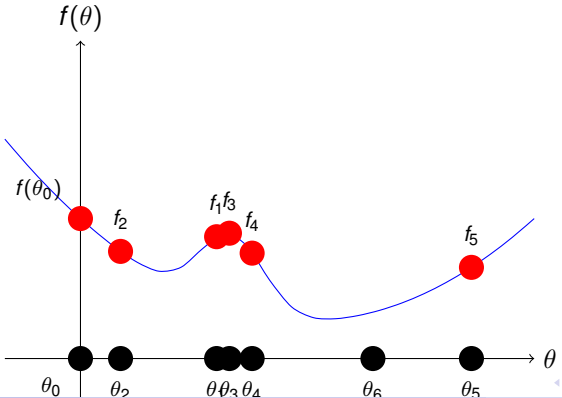

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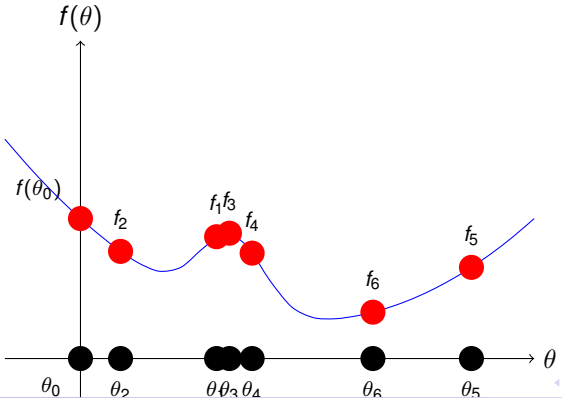

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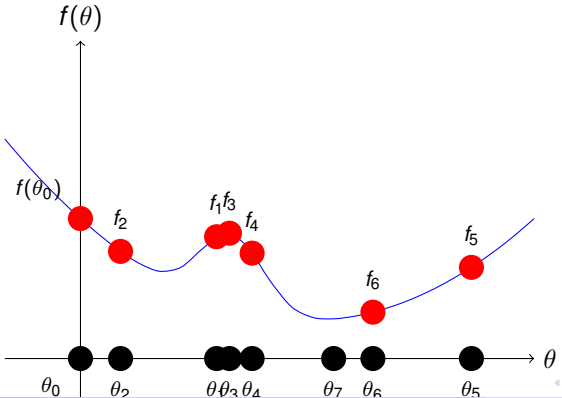

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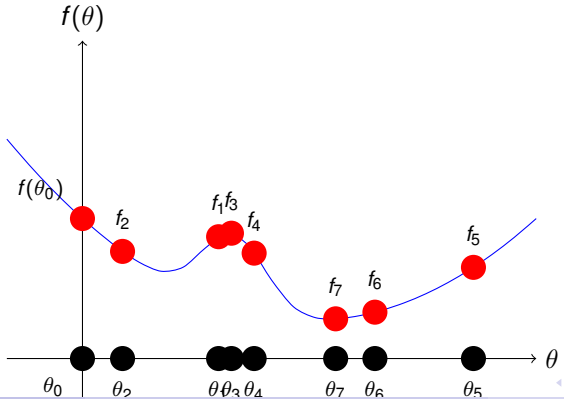

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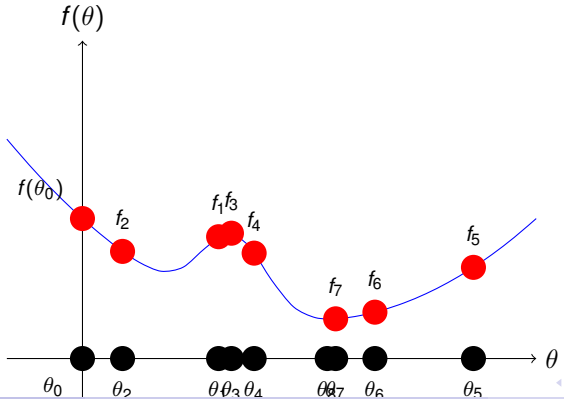

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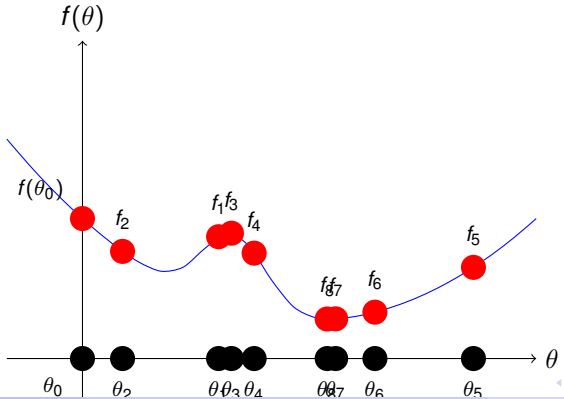

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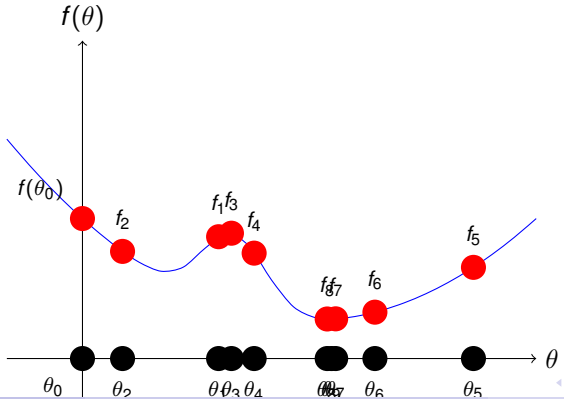

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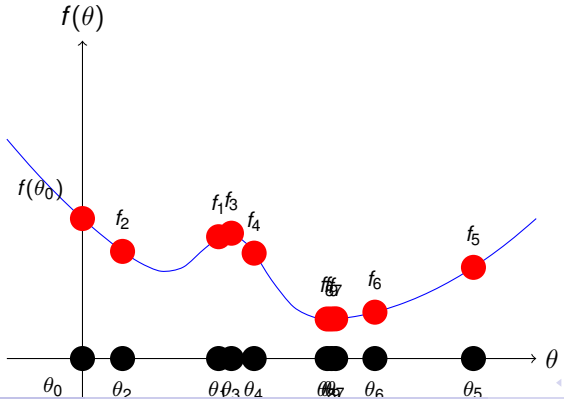

# Non-convexité - À retenir

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- Hormis ces deux points, intuition en 1D complètement fausse

# Non-convexité en haute dimension

- Plateaux
- Points-selle [Dau+14; AHV17]
- Peu de mauvais minima locaux, beaucoup se ressemblent [Cho+15]

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- Rapide (en temps de calcul)
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- Gère bien le conditionnement
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- Aucun algorithme ne fait tout ça aujourd'hui

# Idée 1: Utiliser plusieurs pas de gradient

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- De nombreuses variantes existent [DHS11; KB14]

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- Cette méthode s'appelle Batch-Norm [IS15]

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  - On veut réduire $\alpha$ dans les directions courbées
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- Ces deux propriétés demandent de la mémoire.

# Momentum [Nes83]

- Momentum garde une mémoire des gradients passés
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- Ajout d'un deuxième hyperparamètre.

# Généralisation

- But de l'apprentissage est de bien généraliser
- Aucune différence dans le monde convexe
- Optimiseur a un impact sur la généralisation dans les réseaux profonds
- Raison encore peu claire

# Robustesse: un point essentiel

- Pas les mêmes qualités nécessaires pour un entraînement unique et pour un entraînement régulier
- La difficulté de réglage est un point critique qui n'apparaît pas dans les courbes
- Attention aux modifications du pas de gradient pendant l'optimisation [TH12]
- L'erreur qui vous importe est l'erreur de généralisation, pas d'entraînement

Merci !

nlr@google.com

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